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I. *The Constitution of Mercury Lines examined by an Echelon Grating and a Lummer-Gehrcke Plate.* By Prof.

H. NAGAOKA *and* T. TAKAMINE, *Imperial University, Tokyo.*

RECEIVED SEPTEMBER 4, 1912. READ OCTOBER 25, 1912.

§ 1. *Object of the Investigation.*—The constitution of mercury lines has been investigated by different physicists, but the results are generally not in good agreement. It may be due, on the one hand, to the imperfection of the optical instruments, and, on the other, to the nature of the source of light.

With an echelon spectroscope, the presence of ghosts can hardly be avoided, and the order of the spectrum, for satellites at some distance from the principal line, is extremely ambiguous, due to overlapping.

Although the instrument suffers from these two defects, the intensity of light is far superior to that in other interferential apparatus, as it concentrates the light mostly on one or two orders of spectra. An interesting Paper by Stansfield and Walmsley* sheds some light on the nature of the ghosts to be observed in an echelon grating, but the elimination of the ghosts can be easily effected by crossing the spectra according to the method of Gehrcke and von Baeyer,† which is analogous to that of crossed prisms first used by Kundt in investigating anomalous dispersion. The echelon may be crossed by a grating, or by another echelon, or by a Lummer-Gehrcke plate. We followed the last method to clear some mists, which still hang round the nature of the satellites, and that for reasons which will be explained afterwards.

The intensity of the satellites has not yet been accurately measured nor has the regularity in the position of the satellites been well ascertained. According to our investigation, the intensity as well as the position of the satellites are not altogether irregular ; this fact will be of special interest to those who wish to unravel the secrets of atomic structure.

§ 2. *Echelon Grating.*—The echelon grating and the Lummer-Gehrcke plate used in the present investigation were made by Hilger. The former was of the following dimensions :—

Thickness of the plate	9·350 mm.
Number of plates	35
Steps	1 mm.
Length	32·73 cm.

* Stansfield and Walmsley, "Phil. Mag.," **23**, p. 25, 1912.
† Gehrcke and v. Baeyer, "Ann. d. Phys.," **20**, p. 267, 1906.

The thickness given by Hilger agrees with that obtained by actual measurement on a piece of prism cut out from the same plate used for constructing the echelon,* by means of Abbe's contact micrometer (Dickenmesser) reading to one micron. It was compared with a nickel-steel étalon previously standardised at the Bureau International des Poids et Mesures. The above prism (refracting angle $60° 0' 0{\cdot}9''$) was used in finding the indices of refraction of echelon plate for mercury lines with a spectrometer (diameter of the graduated circle 30 cm., reading to 1″ by microscopes placed diametrically opposite). They can be almost exactly expressed either by means of Cauchy's or Hartmann's formula. It is found on interpolation that the numbers given by Hilger for refractive indices differ slightly in the fourth decimal.

The separation of the spectra of consecutive orders in the position of minimum deviation is given by the formula

$$d\lambda_{\text{max.}} = \frac{\lambda^2}{t\left\{(\mu-1)-\lambda\dfrac{\partial\mu}{\partial\lambda}\right\}}.$$

The value of $d\lambda_{\text{max.}}$ (in Å.U.) and the indices of refraction are as follows :—

λ	5,790·5	5,769·5	5,461·0	4,358·6	4,078·1	4,046·8
μ (15°C.)	1·57456	1·57473	1·57725	1·59181	1·59790	1·59873
$d\lambda_{\text{max.}}$	0·5805	0·5760	0·5090	0·3002	0·2547	0·2497

The homogeneity of the glass, as arranged in the echelon grating, was tested by placing the apparatus between two crossed Nicol prisms on a large Paalzow's polariscope, made by Schmidt and Haensch. The dark field was then dimly lighted, but none of the polarisation band was to be detected, showing that the pile of plates, though not entirely free from strain, was almost homogeneous. That the echelon was slightly strained could at once be proved, since a beam of parallel rays on passing through it did not issue as such, but the pile acted somewhat like a lens. This fact is already well known, so that it is only necessary to note it.

* Lunelund ("Ann. d. Phys.," **34**, p. 505, 1911) has some doubt about the thickness of our echelon. The number quoted by Lunelund may refer to another echelon, of which there are several in Japan.

§ 3. *Lummer-Gehrcke Plate.*—The Lummer plate was of the same kind of glass as that of the echelon grating. The plate was 20 cm. long, 3·5 cm. wide and 1·09116 cm. thick. The last number was determined by Abbe's contact micrometer. The indices of refraction were determined directly by means of Abbe's crystal refractometer, the constant of the instrument being checked by a quartz plate, and also by the prism of the echelon plate, whose indices of refraction were already measured by another instrument of high accuracy. The indices thus found were less by 7 or 8 in the 4th decimal place from those of the echelon plate; they were conformable to Cauchy's as well as to Hartmann's formula.

The value of $d\lambda_{\text{max.}}$ may be calculated in the following manner. The order of the spectrum h is given by

$$h=\frac{2t\sqrt{\mu^2-\sin^2 i}}{\lambda}, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (1)$$

where t is the thickness and i the angle of exit.

Differentiating it with respect to λ,

$$h^2\lambda=4t^2\left(\mu\frac{\partial\mu}{\lambda}-\frac{\sin 2i}{2}\frac{\partial i}{\partial\lambda}\right). \quad . \quad . \quad . \quad . \quad (2)$$

Since h is of the order 5×10^4 in the present experiment, we get the equation of finite difference

$$h\lambda^2 \,.\, \Delta h=-2t^2 \sin 2i \,.\, \Delta i. \quad . \quad . \quad . \quad . \quad . \quad (3)$$

Equation (2) gives an approximate relation

$$2t^2 \sin 2i \,.\, \Delta i=\left(h^2\lambda-4t^2\mu\frac{\partial\mu}{\partial\lambda}\right)\Delta\lambda. \quad . \quad . \quad . \quad (4)$$

Hence, by putting $\Delta h=1$, we have $\Delta\lambda=d\lambda_{\text{max.}}$, so that

$$d\lambda_{\text{max.}}=-\frac{\lambda^2}{h\lambda-\dfrac{4t^2\mu}{h}\dfrac{\partial\mu}{\partial\lambda}}. \quad . \quad . \quad . \quad . \quad . \quad (5)$$

The importance of introducing the correction for dispersion was first recognised by v. Baeyer.*

* v. Baeyer, "Verh. d. Deutsch. Phys. Ges.," **10**, p. 733, 1908.

For grazing exit

$$h_0 = \frac{2t\sqrt{\mu^2-1}}{\lambda}, \quad \ldots \ldots \quad (1')$$

and the corresponding $d\lambda_{max.}$ (in Å.U.) is as follows :—

λ	5,790	5,769	5,461	4,359	4,078	4,047
h_0	45,820	46,000	48,710	61,980	66,720	67,200
$d\lambda_{max.}$	0·12090	0·11996	0·10659	0·06466	0·05596	0·05450

When the angle of exit i deviates from 90° by 2° or 3°, it is necessary to introduce a small correction to $d\lambda_{max.}$, obtained for $i=90°$. If the angle of exit cannot be measured directly, it is approximately calculated by using (3).

$$\text{Sin } 2i = -\frac{h_0\lambda^2}{2t^2} \cdot \frac{\Delta h}{\Delta i}. \quad \ldots \ldots \quad (3')$$

It is generally sufficient to take $\Delta h=10$ or 15, find the corresponding Δi, and then calculate the correction. This is easily done on a photograph.

In using the plate and the echelon gratings care was taken to protect them from changes of temperature by enclosing them in wooden boxes which were lined thickly with cork plate. Nobody remained in the room while the photographic plate was exposed.

The lamp used in the present experiment was mostly of Arons-Lummer type, fed by direct current of 10 amperes at 30 volts, and cooled by water current. Sometimes a Heraeus quartz lamp was used, but generally the lines were more distinct with the Lummer lamp, especially when it was well cooled and placed under low voltage. The photographs as well as visual observation show that the lines are better defined when the line of sight is parallel to the arc than when it is transverse.

This is well exemplified in the satellite next to the principal line of 5,461 (Fig. 17).

In the present investigation we did not enter into experiment on the changes in lines caused by introducing different gases into the lamp, as was recently done by Wendt.*

The plate was provided with a right-angled prism whose section is an isosceles triangle, as in the original form used by

* Wendt, "Ann. d. Phys.," 37, p. 535, 1912.

Lummer and Gehrcke; but it was found more convenient to change it into another, whose section is a right-angled triangle with one angle of about 22·5°, so that the plate can be used *à vision directe*.

The resolving power of the plate and of the echelon grating was nearly the same, and for wave-length of 0·5μ it is 435,000 for the echelon grating and 400,000 for the plate.

§ 4. *Arrangement.*—In order to eliminate the ghosts, which inevitably appear in echelon gratings, the spectrum was crossed by the Lummer plate after the method of Gehrcke and v. Baeyer. For this purpose the plate was placed horizontally and the echelon grating in such a position that the spectrum lines were vertical. The arrangement is shown in Fig. 1.

The light emerging from the echelon grating was made parallel by means of Zeiss micro-planars (focal lengths 7 cm., 5 cm., 3·5 cm.), and made to fall on the vertical face of the prism attached to the Lummer plate. The interference points

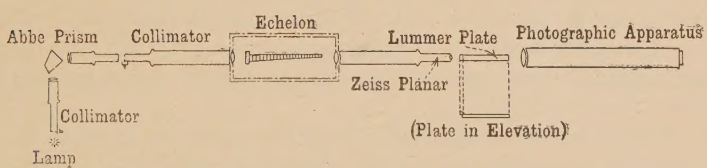

Fig. 1.

were photographed by means of Goerz anastigmat (focal length 21 cm., aperture 3 cm.), or by photographic lenses of 10 cm. aperture, having focal lengths of 70 cm. and 123 cm. respectively.

These micro-planars and photographic lenses were chosen so as to meet the special object of each experiment.

In order to obtain the photographs for the measurement of positions of satellites, and for testing the position of ghosts, lenses of long foci were generally used; while for the purpose of examining the number of satellites and for photographing the lines with short exposure, a micro-planar of long focus ($F=7·0$) was combined with the Goerz anastigmat.

On this account the photographs given in this Paper are of various magnifications.

The Goerz lens was attached to a photo-theodolite by Günther and Tegetmeyer, as there was advantage in finding the angle of exit directly from the readings of the divided circle. In order to find the interference points of faint lines it was

necessary to use lenses of short focal length, but for the determination of the deviation of satellites from the principal line, photographs obtained by lenses of longer focal lengths were used to secure more accuracy without increasing the magnifying power of the micrometer with which the relative position of the points was measured. For the latter purpose the slit in front of the echelon grating was generally opened to a width of about 0·04 mm., so that the crossed images appeared as short lines instead of points on the photographic plate. In this manner the lines of satellites could be placed between two spider lines of the micrometer, and its relative position exactly determined. When the points were crowded it was sometimes necessary to work with a slit only 0·01 mm. wide. In spite of the small quantity of light which was able to pass through the echelon as well as the plate, the exposure did not exceed five or six hours even in the very insensitive part of the spectrum.

For the photography of yellow and green lines, "panchromatic spectrum plates" of "Wratten and Wainwright" were used, and for lines in the violet region we used "Wratten process plate." Care was taken to develop and fix the plates under the same conditions.

The focussing of the lens was rather a tedious process, as it required sometimes more than a dozen photographs to obtain a sharp focus, especially for lines in the almost invisible violet region. But in such a region of the spectrum exposure did not last more than 10 or 20 minutes before we can obtain a fairly good image. Thus we can utilise the bright image given by the echelon in shortening the time of exposure of photographic plates.

§ 5. *Crossed Spectra.*—We tried different methods of crossing the spectra. The echelon spectrum was crossed with that obtained by a metallic plane grating ruled on a Rowland engine; but, owing to the faintness of light and the low resolving power of the grating, the results were by no means comparable with those obtained by the combination of the echelon and the Lummer plate. The same remark applies to the crossed spectra of two echelon gratings. The echelon above described was crossed with another belonging to the Tōkyō Higher Normal School, having a resolving power of 140,000.* As it was necessary to work with very small slits,

* The instrument was placed at our disposal through the courtesy of Prof. Noda, to whom our best thanks are due.

the quantity of light was generally insufficient to show the details of the spectra, though it was much superior to the crossed spectra of echelon and ordinary grating.

§ 6. *Advantages of Crossing the Spectra.*—In all of our observations the angle of exit was nearly equal to 90°, so that by putting

$$i=90°-\alpha,$$

where α is small, the equation giving the order of spectrum becomes

$$h^2\lambda^2=4t^2\{(\mu^2-1)+\alpha^2\},$$

neglecting higher powers of α.

Putting $\lambda=\lambda_0+\delta\lambda$, where λ_0 is the wave-length of the reference line and $\delta\lambda$ the deviation of the wave-length of satellites from it,

$$2h\lambda_0\delta\lambda=\{8t^2(\mu^2-1)-h^2\lambda_0^2\}+\alpha^2.$$

In the crossed spectra given by the echelon grating and Lummer plate we may conveniently take for abscissæ the distances of the satellites from the principal line on the echelon spectrum, and hence proportional to $\delta\lambda$, and ordinates as proportional to deviation α from grazing exit.

Consequently the locus of the interference points for the same value of h and for different satellites must lie on a parabola given by the equation of the form

$$y^2-a^2=bx,$$

where a, b are constants depending on h. For consecutive values of h the parabolas cut the x axis at nearly equidistant intervals.

Thus, by tracing the curve, we are able to arrange the interference points according to different values of h. This method is sometimes of great assistance in discriminating the position of satellites, especially when they are crowded together, as observed with the echelon or the plate only. In all of our measurements we plotted the interference points from the readings of micrometer with respect to x and y axes, in order to fix the order of the spectrum.

Figs. 8, 15, 24 show at a glance the efficacy of the method, especially for 5,790 and 4,359, in which points belonging to different orders of the spectrum are mixed together.

In order to evaluate the relative positions of the satellites it is necessary to refer them to a line which is sharply defined.

It is customary to refer them to the principal line, which is generally broad and has hazy boundaries. It is, therefore, inaccurate to take the principal line as the line of reference, and the discrepancies among different observers seem sometimes to be attributable to the uncertainty in the position of the principal line. We have in most cases used a well-defined satellite as the reference line (indicated by an asterisk in the tables), and afterwards reduced it to the principal line.

With the plate, numerous orders of spectra can be measured with a micrometer, while only a single spectrum can be placed under test with the echelon spectroscope. Moreover, the optical errors are simpler in the plate. Consequently, the result is far more accurate than that with the echelon. In all the calculations which will be made hereafter we only take the values obtained by the plate into account.

§ 7. *Interference Points.*—Owing to the limit in the resolving power of the optical instruments, the fine structure of some lines cannot be exactly known. Although some lines cannot be separated, we find that interference points due to the combination of the echelon grating with Lummer plate sometimes present a singular appearance. When the point is due to the intersection of simple strong lines it has a head and a tail, if we may so call the tapered ends of the point, which are in the direction of the tangent to the parabola joining the interference points of the same order. This is almost always exemplified in the photographs of the crossed spectra. With some points, however, the appearance is greatly modified. The head and tail, instead of being simple, sometimes present a complicated appearance. This is observed also when there is a weak satellite near a strong line, especially when the photograph is of long exposure. By this analogy we may, perhaps, be able to draw some inference as to whether the line is simple or not from the appearance of the points, although the lines may not be distinctly separated. The same remark applies to the examination of the blackened line of photographic plates by a thermopile (§ 8).

When there is a ghost in the neighbourhood of a strong line, the interference points taper towards the ghost in the direction of the x axis in the diagram of interference points when it is due to the echelon, and in the y direction when it is attributable to the plate. An example of the former case is seen in the crossed spectra of 4,078 (Fig. 29, Plate VII.); faint traces of the ghosts

due to the plate were only noticed in the line 5,461. The plate was, therefore, nearly free from ghosts. A number of ghosts were found in the echelon spectrum of the lines 5,461, 4,359 and 4,047, which may be seen in the photographs given in Figs. 16, 27 and 33 (Plates III., VI. and VIII.).

§ 8. *Relative Intensity.*—In spite of the numerous investigations on the position of satellites of mercury lines, very little has been done on the relative intensities of the satellites. The exact photometry of these lines would be a matter of great difficulty, but as the deviations of satellites are only a fraction of Ångstrom unit in most cases, we may treat it as monochromatic, and hence assume the constant of the Schwarzschild's* law of blackening of photographic plates to be the same for the principal line and the satellites. Under this assumption, we may measure the relative intensities when the blackenings of the lines to be compared lie within the normal regions of the curves of blackening (Schwärzungskurve).

For this purpose, it was, in the first place, necessary to photograph the echelon spectrum under several different degrees of exposures. Firstly, normal exposure for the principal line and strong satellites, for which the faint lines are mostly in the condition of under-exposure ; secondly, normal exposure for satellites of strong and mean intensity, by which the principal line is over-exposed ; thirdly, normal exposure for fainter lines, and so on. By the combination of these plates we may arrive at an approximate value of the relative intensities by successive comparisons. The echelon spectra of two successive orders were generally photographed in the position of minimum deviation, by which the satellites were all seen between the images of the principal line, which were equally intense on the plate.

In order to secure the constancy of the current and voltage, the Heraeus mercury lamp was found preferable to an Arons-Lummer lamp. Both lamps were used in the present experiment. For the measurement of the relative intensity the spectrum of the echelon grating was photographed without the interposition of the Lummer plate. The space between the consecutive orders of the spectrum ranged from 3 mm. to 9 mm. on the photographic plate. This plate was attached to a micrometer and placed almost in contact with a slit (17 mm.

* Schwarzschild, " Publik. d. Kuffner. Sternwarte," 5, 1900.

long, 0·05mm. wide), so that the lines were parallel to it. The arrangement is shown in the following figure (Fig. 2).

The light from a Nernst lamp (110 volt, 1 ampere) was made parallel by a quartz lens and passed the plate at normal incidence. The current feeding the lamp was carefully adjusted during the observation. A Rubens' thermopile, consisting of 24 junctions of iron and constantan wire, was placed behind the slit, and the light passing through the slit fell on the junctions, which were arranged in a straight line. The deflection of a D'Arsonval galvanometer, connected with the pile, was read when a successive displacement of 0·05 mm. was given to the plate.

There was great difficulty in keeping the zero of the galvanometer unchanged, as the metal case containing the thermo-

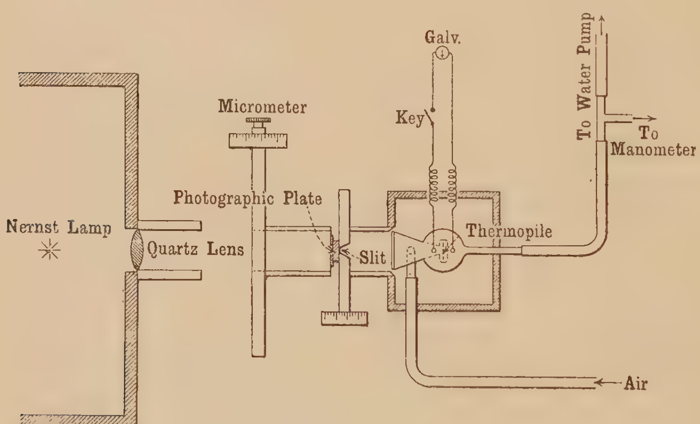

Fig. 2.

pile got gradually heated, so that the temperature of the cold junction was gradually rising. To avoid this inconvenience, the metal case of the pile was closed in the front by a window of thin microscope cover glass, and was placed within a wooden box filled with cotton. A slow current of *dry* and dust-free air, whose temperature was made constant by passing it through a long lead tube immersed in a large water tank, was maintained through the metal case by a water pump, the pressure difference being 3 mm. of water. By this means the zero of the galvanometer remained nearly constant. Of course, it was necessary to pass the air current for several hours before starting the observation.

Usually it took seven or eight hours to examine the satellites of a single line.

Objections may be raised against this method of measuring the relative intensities by the blackening of the photographic plate. Fortunately the deviations of satellites are all very small, so that the question as to the validity of assuming the constant of Schwarzschild's law to be the same for different wave-lengths does not come into the problem now under discussion. The effect of diffraction and scattering due to the

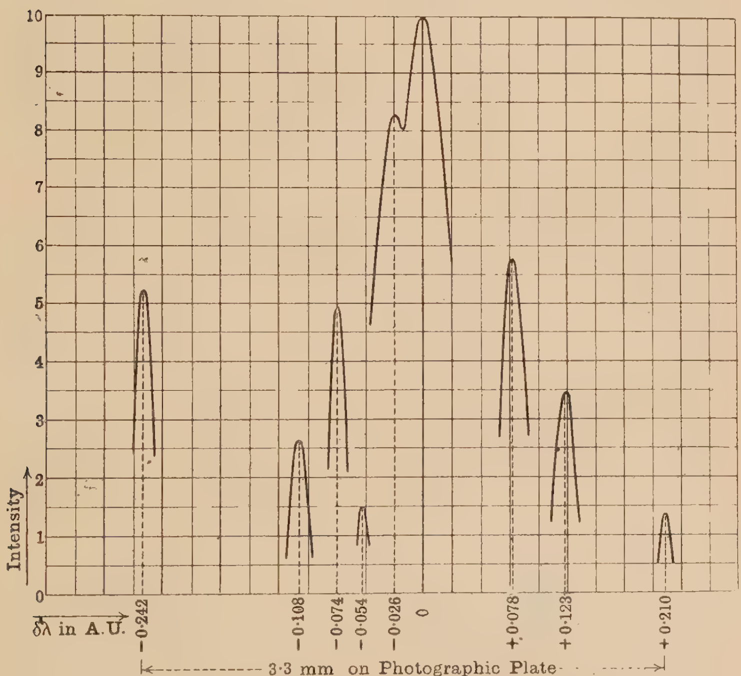

Fig. 3.—Intensities of the Satellites of λ 5,461.

line as well as the slit was entirely neglected. This may not be negligibly small. The absorption of light by the photographic plate and the film, as well as the sensibility of the film, are considered to be uniform throughout the length of the plate measured. These are some of the defects attached to the above method, but to the first approximation the galvanometer indications may doubtless be taken as a measure of relative intensity. The accompanying figure shows how the

galvanometer is affected as the plate is made to pass over the slit. We shall afterwards state that the intensities in some groups of satellites are regularly distributed.

The existence of the ghosts in the photographs taken with the echelon grating alone causes disturbance in the blackening due to the real line, especially when the ghost is very near to the line. In addition to this, when a feeble line is in the neighbourhood of a strong one, the former is almost masked by the latter, but we occasionally notice a slight protuberance which disturbs the smoothness of the curves traced by plotting the galvanometer deflections. That these slight protuberances were not due to some accidental causes, such as might come from the change of intensity of the lamp, or that of the velocity of the air current, &c., has been verified by the fact that when we moved the micrometer backwards exactly the same protuberance was seen in the reverse way.

Although the intensity measurement was carried out on the photographs of the six lines of mercury, it was the green line 5,461 alone which was examined in detail by taking a number of photographs under different exposures. For the other lines the photographs were taken only in two or three different exposures, so that the results were not so accurate as in that of 5,461. On this account we have given the values of relative intensities to two places of decimals for 5,461 only.

It may be remarked that, when a faint satellite was very close to a strong line, there was much difficulty in determining the intensity of the former owing to the diffusion of photo-chemical action caused by the latter.

§ 9. *Satellites.* 5,790.—Of the different lines examined in the present experiment, none presents such complex structure as the line 5,790. There are numerous lines in 5,461 and 4,359, but the distribution, as they appear with the echelon only or crossed by the plate, is tolerably simple. With the yellow line 5,790 the complexity is due to the existence of two satellites, both at about $-1 \cdot 0$ Å.U. from the principal line. The stronger line of the two was noticed by several previous investigators, and both lines were directly photographed with a Michelson grating by Gale and Lemon.* In the course of our investigation the stronger satellite was easily photographed by a Rowland concave grating (radius $10\frac{1}{2}$ ft., 14,438 lines to the inch), of which Fig. 5 shows its position with respect to the

* Gale and Lemon, "Astrophys. Jour.," **31**, p. 78, 1910.

principal 5,790 and the neighbouring line 5,769, already in the first order spectrum. The distance of the satellite from the principal is about one-twentieth of the distance between the two strong lines. This line and its companion appear in the spectrum of the echelon grating mixed with other satellites (Fig. 6), which lie very near together, and are of the same order as the principal, while the said satellite is two orders higher. To discriminate the lines from other faint lines it is necessary to cross the echelon with the plate (Fig. 7). By measuring the position of the interference points by means of a micrometer, and joining the points belonging to different satellites by parabolas, which form loci of points for the same orders of spectra, we easily find that there are distinctly two series of points which belong to different orders of spectra from other neighbouring points. The result of micrometric measurements is given in Fig. 8, and the copy of the original in Fig. 7 (Plate I.). The appearance of the echelon spectrum with the respective positions of the other satellites are given below the diagram of the interference points. The ghost is marked with the letter G. They are all blended together, so that it seems impossible to discriminate the distribution of satellites with the echelon grating alone, as the accompanying diagram of the echelon spectrum will show (Fig. 8).

Although the crossed spectra show beyond doubt the positions of these satellites, we tried to bring fresh evidence as regards the positions of the satellites in the echelon spectrum.

Instead of using the spectrum given by the Abbe prism in the echelon apparatus as constructed by Hilger, we have projected the line given by the concave grating, in which the stronger satellite -1 Å.U. was distinctly separated from the principal line, on the slit in front of the echelon grating, so that it can be included or excluded in obtaining the echelon spectrum. The spectra, with and without the said satellites, are given in Figs. 9 (*a*) and 9 (*b*) (Plate II.) respectively. It shows that the third strong line from the left is due to the satellite about $-1{\cdot}0$ Å.U. distant. The fourth line is a little displaced in relative position, which shows that the neighbouring satellite is almost coincident with the strong line. It will be worthy of remark that where there is some doubt as to the legitimacy of the position of satellite we may bring in the aid of the instrument of high dispersion, and by process of elimination arrive at a correct result.

A discussion was raised by Gmelin* as to the existence of the satellite +0·164. By a method which is quite analogous to

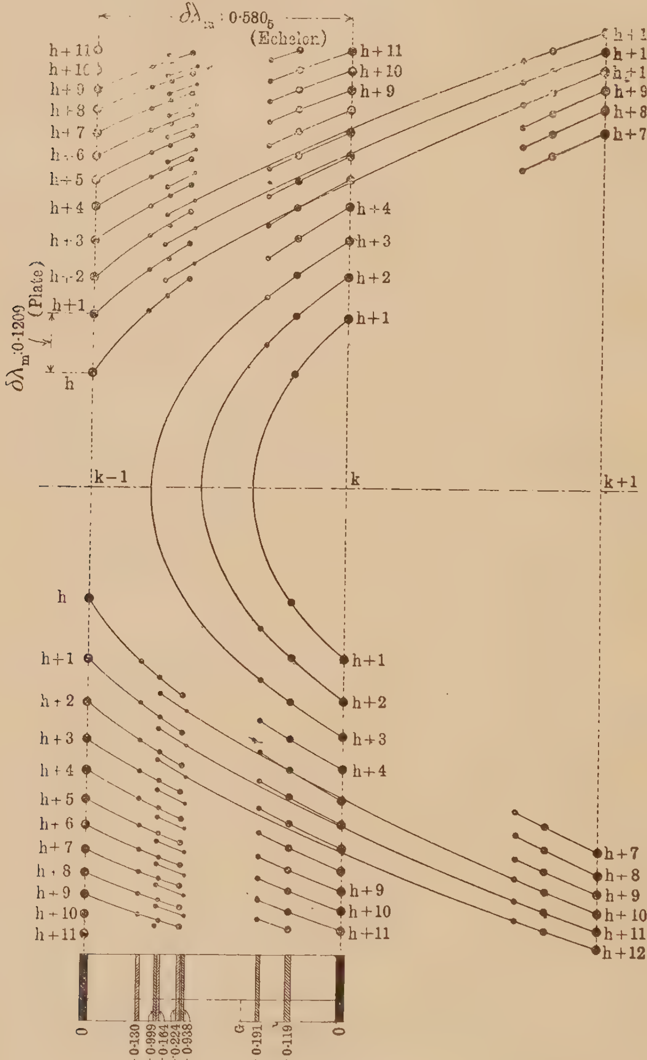

FIG. 8.

that of crossed spectra, he showed that, in place of the said line, there is a satellite −0·374. With the echelon here used,

* Gmelin, "Ann. d. Phys.," **33**, p. 17, 1910.

and also with the crossed plate, the line falls in the neighbourhood of $-0{\cdot}938$ and $+0{\cdot}224$. Of the numerous photographs which have been taken with different micro-planars and photographic lenses it was difficult to see distinctly the exact position of the said satellite. There are several indirect evidences for its existence, but we have not given it in the figure as its existence cannot be directly ascertained.

As to the faint line $+0{\cdot}164$, which appears distinctly in our crossed spectra, and has been measured by Janicki,* Galitzin,† and Lunelund,‡ there is not a least doubt of its existence. It may be due to the accidental coincidence of the line with the alleged $-0{\cdot}374$ in the echelon spectra of the above-mentioned observers that the line was cancelled by Gmelin. It is also very curious that $-0{\cdot}938$, which first appears in the observations of Gale and Lemon, had not been noticed previously. In our case it is altogether impossible to discriminate it from $+0{\cdot}224$ in the echelon spectrum ; but the interposition of the Lummer plate places its existence beyond doubt, as illustrated in the diagram of the interference points. The order of plate spectrum for $0{\cdot}938$ is much higher than that for the neighbouring point $+0{\cdot}224$. This is of special interest, showing how the crossed spectra can sometimes analyse the coincidence of several lines in a single echelon spectra by separating them into different interference points.

Instead of giving a table of the results of different observers, Fig. 10 will present at a glance the coincidence as well as the discrepancy of different measurements. The thickness of the line is drawn proportional to the intensity.

It is to be noticed that, the principal line being wide and diffuse, the mean point is difficult to determine ; the consequence is that the distribution of the satellites by one observer is one-sided compared to that of the other. The results of our measurement are tabulated below :—

Echelon ..	−0·996	−0·931	−0·196	−0·129	§0	+0·132	+0·161	+0·217
Plate......	−0·999	−0·938	−0·191	−0·119	§0	+0·130	+0·164	+0·224
Intensity	2	1	2	6	10	3	2	4

* Janicki, "Ann. d. Phys.," **19**, p. 36, 1906.

† Galitzin, "Bull. de l'Acad. Imp. d. Sc. de St. Petersburg," p. 159, 1907.

‡ Lunelund, "Ann. d. Phys.," **34**, 505, 1911.

§ The line marked by an asterisk is the reference line in the determination of $\delta\lambda$.

We consider hereafter the result obtained by the plate as more accurate than that by an echelon, inasmuch as the optical

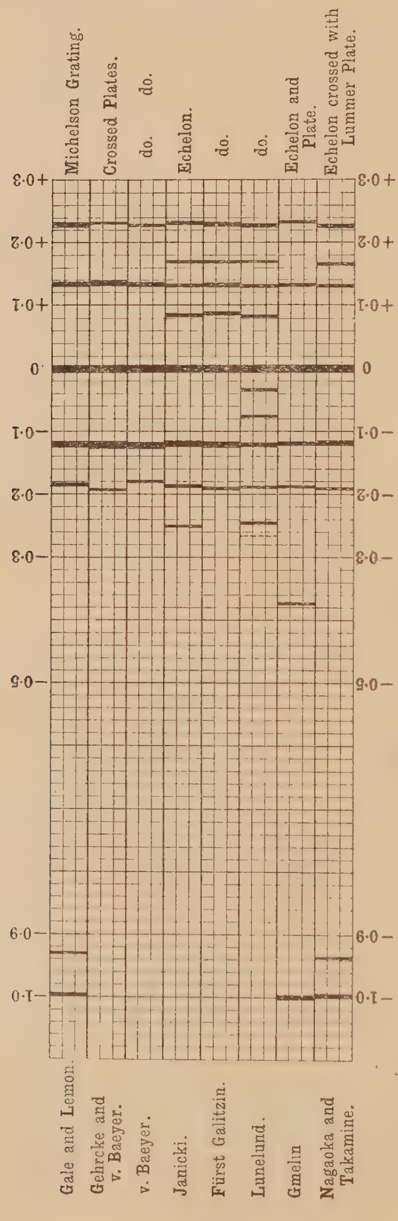

FIG. 10.—STRUCTURE OF λ 5790.

errors attending the former are simpler than the other, and as micrometric measurements can be made on numerous orders of spectra.

Gmelin showed that +0·084 of Janicki and +0·086 of Galitzin are coincident with the satellite −1·0 Å.U. Lunelund recognised that +0·082 of his observation would correspond to the above satellite at the position of

$$-2d\lambda_{\text{max.}}+0{\cdot}082=-1{,}006 \text{ Å.U.},$$

but he had no means of detecting the difference in order from the other satellites.

5,769.—This line is quite simple. It has three satellites, of which two are nearly symmetrical about the principal in position as well as in intensity. The third satellite seems as

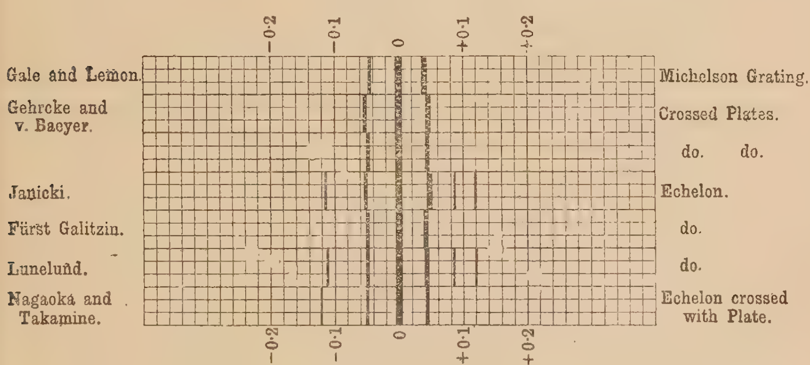

FIG. 13.—STRUCTURE OF λ=5,769.

if it were the ghost of the principal, but micrometric measurement shows distinct deviation from the position which it must occupy if it were a false line due to echelon.

Fig. 11 shows the crossed spectra and Fig. 12 the echelon spectrum.

Our measurements are as follows :—

Echelon............	−0·109(?)	−0·049	*0	0·046
Plate	−0·121	−0·050	*0	0·044
Intensity	1	3	10	3

The lines +0·084 and +0·121 given by Lunelund did not appear in the crossed spectra, and are probably ghosts. The results of various observers are shown diagrammatically in Fig. 13.

5,461.—Much interest attaches to this line, as it is the brightest and the most accurately examined of the lines of mercury. The crossed spectra (Fig. 14, Plate III.), the diagram

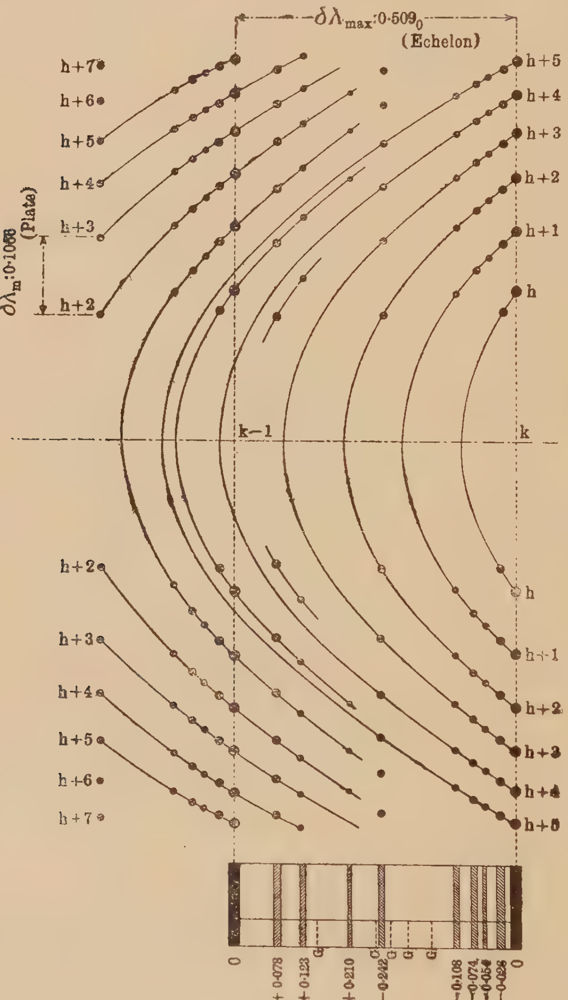

Fig. 15.

of the interference points (Fig. 15) and the echelon spectrum (Fig. 16, Plate III.) show at once the presence of eight satellites, of which the existence can hardly be doubted. In order

to separate the principal line from its nearest companion, —0·026, it is necessary to run the lamp at low voltage, and place the line of sight in the direction of the arc. Also good cooling of the tube is necessary. The photographs of echelon and of crossed spectra taken under these conditions are shown in Figs. 17 and 18 (Plate IV.). By long exposure of the photographic plate we noticed fine ghosts (indicated by G in the diagram annexed to Fig. 15) in the neighbourhood of the line —0·242 and +0·210; these are very much like those observed by Stansfield, who, however, considered the real line —0·054 also as a ghost. The faint line +0·210 is not easy to measure, but its existence is beyond question (Fig. 19, Plate IV.).

The results of different observers are given in the diagram below.

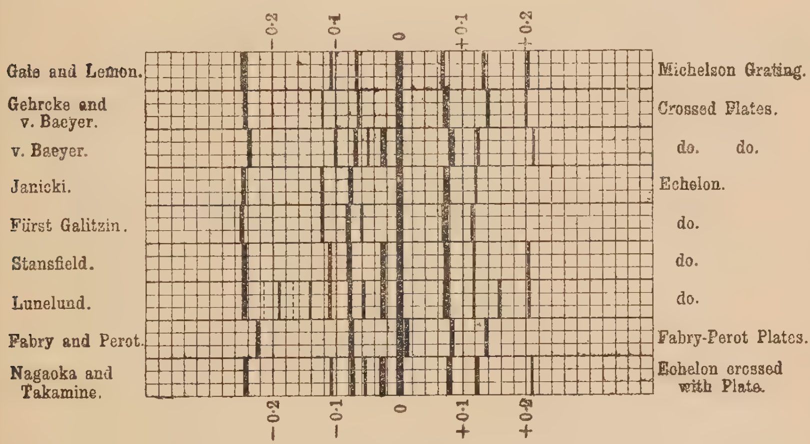

FIG. 20.—STRUCTURE OF λ=5,461.

Our measurements are as follows:—

Echelon ...	*—0·246	—0·113	—0·080	—0·055	—0·026	0	+0·072	+0·118	+0·204
Plate	*—0·242	—0·108	—0·074	—0·054	—0·026	0	+0·078	+0·123	+0·210
Intensity	5·22	2·62	4·92	1·50	8·26	10	5·77	3·47	1·34

These numbers agree very well with those found by v. Baeyer. Several of the lines cited by Lunelund occupy similar positions to the ghosts in our echelon spectrum, and will probably have no real existence.

We shall afterwards see that the intensities of the different satellites are in a regular order.

4,359.—This strong line is accompanied by 10 satellites, so that the crossed spectra are by no means simple ; but as they

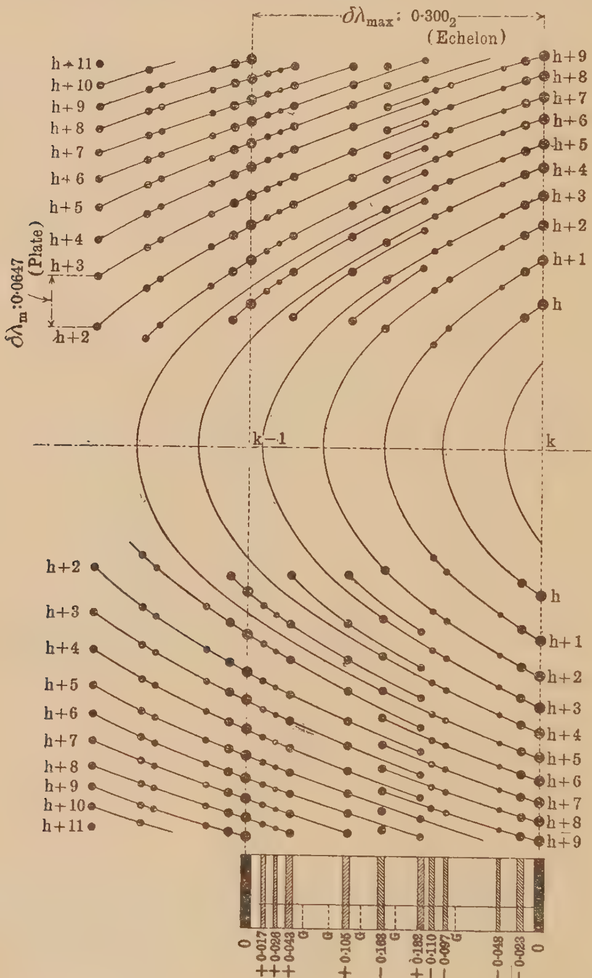

Fig. 24.

are not so widely distant from the principal as in 5,790, the discrimination as to the order and position of the interference points is not so tedious as with the yellow line. The echelon

spectrum is accompanied by five ghosts (G in annexed diagram of Fig. 24), which are so intense that they may be easily mistaken for real lines, had they not been eliminated by crossing with the Lummer plate. More than 100 photographs were taken before we were sure of the existence of the lines. Fig. 21 (Plate V.) shows the copy of an original, Fig. 22 enlarged, Figs. 25 and 26 (Plate VI.) more magnified in the higher orders of the crossed spectra. The photographs of the echelon spectrum, Figs. 23 and 27, indicate how misleading they are if we have only to rely on them without some means of separating the false from the true lines. The diagram below, indicating the results of different observers, shows how unreliable the echelon spectrum is alone.

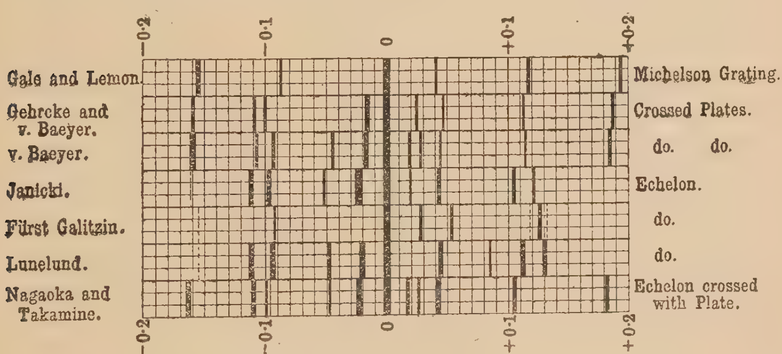

FIG. 28.—STRUCTURE OF λ =4,359.

The following table gives our measurements :—

helon..	−0·161	*−0·108	−0·094	−0·045	−0·019	0	+0·020	+0·032	+0·045	+0·106	+0·183
ate	−0·163	*−0·110	−0·097	−0·048	−0·023	0	+0·017	+0·026	+0·043	+0·105	+0·182
tensity	6	4	3	1	8	10	4	1	5	5	4

The number of lines as well as the positions agree tolerably well with v. Baeyer, who had the command of the highest resolving power of all the experimenters above cited.

The diagrammatical representation (Fig. 28) of the distribution of lines shows a close resemblance with that of the satellites of 5,461.

4,078.—The structure of this line is simple, as shown in the crossed spectra (Fig. 29, Plate VII.) and in the echelon spectrum (Fig. 30, Plate VII.).

The appearance of the latter figure reminds one of the similarity with 5,790, if the line −1·0 Å.U. be eliminated. The deviations among different observers are not so large as in the case of other complex lines. Our measurements are given below :—

Echelon	−0·079	*−0·048	0	+0·029	+0·044	+0·067
Plate	−0·077	*−0·047	0	+0·032	+0·050	+0·076
Intensity.........	4	5	10	3	5	2

It is very singular that there is no discrepancy among different observers as to the existence of five satellites, although the positions differ slightly, as will be seen in Fig. 31.

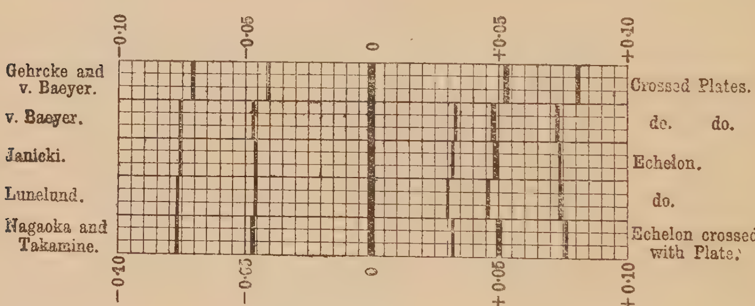

FIG. 31.—STRUCTURE OF λ - 4,078.

4,047.—The echelon spectrum of this line sometimes shows a number of ghosts by long exposure. Fig. 33 gives the photograph taken with 15 minutes' exposure, and Fig. 32 the

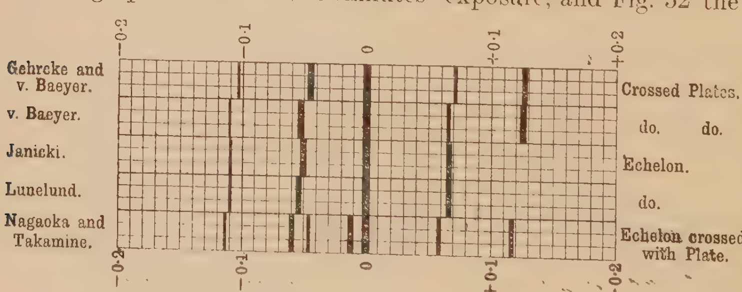

FIG. 35.—STRUCTURE OF λ=4,047.

crossed spectra with a lens of short focus. A lens of long focus gives more detail (Fig. 34). The principal line is found to be

double, and near it is another doublet, which has been often considered as a single broad line. The diagrammatical representation (Fig. 35) of the position of the satellites shows the discrepancies among different observers. Although we have taken photographs with exposures of several hours, we have not been able to establish the existence of the numerous satellites given by Wendt; it appears that they were mostly due to the nature of the lamp.

Our measurements are given below :—

Echelon............	*−0·118	−0·063	−0·049	−0·013	0	+0·060	+0·117
Plate...............	*−0·114	−0·060	−0·046	−0·012	0	+0·059	+0·117
Intensity	3	7	4	8	10	5	5

We have further to remark that the line 4,916 is single; 4,348 and 4,339 are difficult to photograph without the intrusion of the strong line 4,359, so that it is not easy to determine how many satellites accompany them. Probably 4,348 has three, but we tried in vain to have them separated from the lines of 4,359, which inevitably appeared with 4,348 as diffused light.

§ 10. *Regularity.*—One* of us has already pointed out that the distribution of the satellites is not altogether irregular. It was, however, stated with reserve, as the ghosts were not

——	No. of Satellite.	δλ (observed).
Line 5,461	(−5)	−242·2 = −(+2)×2+2·6
	(−4)	−108·3 = (−1)×4+2·3
	(−3)	− 75·0 = −(+1) +2·0
	(−2)	− 55·8 = (−1)×2−2·8
	(−1)	− 26·5
	(+1)	+ 77·0 = −(−1)×3−1·5
	(+2)	+122·4
	(+3)	+209·6 = −(−4)×2+7·0

δλ's are expressed in m.Å.U.

* Nagaoka, "Phys. Zeit.," **10**, p. 609, 1909.

eliminated and were misleading. In the present experiments the ghosts are eliminated, so that we may enter with confidence into the discussion as regards the distribution.

It has already been stated that the $\delta\lambda$ of some satellites are in a simple ratio; a good example is afforded by the satellites of the lines 5,461, 4,078 and 4,047.

Distinguishing the satellites by the number, we find that $\delta\lambda$'s can be expressed as multiples, or sometimes as combinations of $\delta\lambda$ of other satellites as given in the third column of the lower table on previous page.

We obtain the approximate relation

$$\begin{aligned}(-1):(-2):(-3)\text{ or }(+1):(-4)\\ =26{\cdot}5:55{\cdot}8:75{\cdot}0\text{ (or }77{\cdot}0):108{\cdot}3\\ 1\;:\;2\;:\;3\qquad\qquad:\;4.\end{aligned}$$

The ratio is not exact, but it does not seem to be a mere chance coincidence.

—	No. of Satellite.	δλ (observed).
Line 4,078	(−2)	−76·9 = −(+3) −0·7
	(−1)	−46·9 = −(+2) +3·1
	(+1)	+32·0 = −(−2)+(−1)+2·0
	(+2)	+50·0 = −(−1) +3·1
	(+3)	+76·2 = −(−2) −0·7

Lines (−2) and (+3) are symmetrical about the principal; (−1) and (+2) are nearly so; (+1) is given by the difference between (−2) and (−1).

—	No. of Satellite.	δλ (observed).
Line 4,047	(−4)	−114·1 = −(+2) +2·6
	(−3)	− 60·1 = −(+1) −1·6
	(−2)	− 46·4 = −(+1)−(−1)−0·3
	(−1)	− 12·4 = −(+1)−(−2)−0·3
	(+1)	+ 58·5 = −(−3) −1·6
	(+2)	+116·7 = −(−4) +2·6

The approximate symmetry between (-4) and $(+2)$, (-3) and $(+2)$ is at once evident, and $(+2)=2\times(+1)$.

If we investigate the structure of other lines we may, perhaps, find a similar relation; we believe that such a relation is not peculiar to mercury lines only, but is to be found in lines of other elements, of which manganese, investigated by Janicki, is one instance.* Examples of the symmetrical positions of the satellites with respect to the principal line are to be seen in Figs. 10, 13, 20, 28, 31 and 35. This symmetry of position is not, however, always attended with that of intensity.

In addition to this, the character of the distribution of lines is similar, especially in 5,461 and 4,359, which belong to the second subordinate series. In both of these lines the principal line is accompanied by a strong satellite on the side towards the violet, which is very near it. For these two satellites the difference in frequency from their respective principal lines is nearly the same, which is also characteristic of some alkaline elements.

Thus for
$$5{,}461 : \frac{\delta\lambda}{\lambda^2}=88\times10^{-11},$$
$$4{,}359 : \quad ,, \quad =89 \quad ,,$$
$$4{,}047 : \quad ,, \quad =76 \quad ,,$$

The last line, 4,047, shows some deviation, but as the position of the neighbouring satellite is so near that it is hardly resolved by the instruments at our command, we must wait for a more accurate determination with instruments of greater resolution.

Another characteristic is that the numbers $\frac{\delta\lambda}{\lambda}$ have values common to several of the satellites in different lines, and are multiples of those preceding them, so that some numbers occur oftener than others, if we construct a table of $\frac{\delta\lambda}{\lambda}$ for different lines.

The following table† covers all the values of $\frac{\delta\lambda}{\lambda}$ for satellites found in the present experiment. For 4,348 the mean of different observers were assumed.

* Janicki, "Ann. d. Phys.," **29**, p. 833, 1909.

† In constructing the above table only $\delta\lambda$'s found from the plate were used; sometimes the fourth decimal was taken into account.

λ.	$\frac{\delta\lambda}{\lambda}\times 10^7$.
4,047	31
4,359	40
5,461	−49
4,359	−53
4,359	60
4,047	2×31=62
5,769	77
4,078	78
4,359	2×40=80
5,769	−87
5,461	102
4,359	98
5,461	2×49=− 98
4,359	2×53=−106
4,359	110
4,348	−110
4,348	115
4,078	−115
4,047	−115

λ.	$\frac{\delta\lambda}{\lambda}\times 10^7$.
4,078	122
4,359	2×60=120
4,359	3×40=120
4,047	4×31=124
5,461	−138
5,461	141
5,461	−3×49=−147
4,047	145
4,047	−149
4,348	186
4,078	187
4,078	−188
5,461	−198
5,461	−4× 49=−196
5,461	2×102=−204
4,359	2× 98=−196
5,790	−206
5,769	−210
5,790	223
5,461	224
4,359	−222
4,359	2×110=−220
4,348	2×110=−220
4,359	240
4,359	4× 60=240

λ.	$\frac{\delta\lambda}{\lambda}\times 10^7$.
4,359	−253
5,790	283
4,047	−282
4,047	289
4,047	2×145=290
5,790	−330
4,359	3×110=330
4,358	3×110=330
4,359	−374
5,461	383
5,790	386
4,348	2×186=372
4,078	2×187=374
4,359	416
5,790	2×206=−412
5,769	2×210=−420
5,461	3×138=−414
5,461	444
5,461	2×224=448
4,359	2×222=−444
4,359	−4×110=−440
4,348	−4×110=−440
5,790	−1,620
5,790	−1,725

It will be seen that some of these numbers are remarkably coincident in several lines, and some of the numbers are multiples of others.

Interpreted in the light of Doppler's principle, the quantity $\pm\frac{\delta\lambda \cdot c}{2\lambda}$ gives the velocity u of the approach or recession, c denoting the velocity of light. The numbers above obtained make u range from several hundred metres to many thousand metres per second. But this is simply a suggestion, and we do not mean that the above quantity is due to Doppler effect. The multiplicity of the above numbers may probably be due to the number of electronic charges which form the centres of light vibration. It would, however, be premature to speculate upon any theory which will explain the facts here described.

It may be suggested that the change of frequency, which is proportional to $\frac{\delta\lambda}{\lambda^2}$, will have the same property as $\frac{\delta\lambda}{\lambda}$; a table of $\frac{\delta\lambda}{\lambda^2}$ was therefore constructed, but no inference could be drawn from it, except the one already mentioned with respect to the companion of the principal line.

The intensities of many satellites seem to follow a simple law. A glance at Fig. 3, representing the intensities of green lines referred to the principal line, will show that they decrease proportionally to $\delta\lambda$ from the principal line for the satellites $-0{\cdot}026$, $-0{\cdot}074$, $-0{\cdot}108$ on the negative side, and for $+0{\cdot}078$ and $+0{\cdot}123$ on the positive side. Again, the vertices of the intensity curves for $-0{\cdot}242$, $-0{\cdot}108$, $-0{\cdot}054$ lie on a straight line, which makes us doubt if these lines are not the satellites of the strong line $-0{\cdot}242$. Only the satellite $+0{\cdot}210$ does not fit in these lines (Fig. 36, p. 28).

It must not, however, be forgotten that the ghosts in an echelon spectrum disturb the intensity of the neighbouring line to some extent.

The law of the proportionality of intensity to $\delta\lambda$ is also indicated in the intensity diagram for 4,359, 4,078 and 4,047; due to the complex structure of 5,790, it is difficult to draw any inference, as also for 5,769, in which the satellites are too few in number.

With the line 4,359 the intensity curve for $-0{\cdot}023$ and $-0{\cdot}097$ lies on a line with the principal, and also for $-0{\cdot}163$, $-0{\cdot}110$ and $-0{\cdot}048$. Perhaps the latter lines are satellites of $-0{\cdot}163$. On the positive side, $+0{\cdot}017$ and $+0{\cdot}026$ lie on a line with the principal.

With the line 4,078, $+0{\cdot}050$ and $+0{\cdot}076$ form one line with the principal, while the last line may also be grouped with $-0{\cdot}047$ and $+0{\cdot}032$. With 4,047, —only $0{\cdot}012$ and $-0{\cdot}046$ form a group with the principal line.

We have further to remark that in all of these lines two satellites of almost equal intensity, which is nearly half of that of the corresponding principal line, are always to be found—*i.e.*, $-0{\cdot}242$ and $-0{\cdot}074$ in 5,461, $+0{\cdot}043$ and $+0{\cdot}105$ in 4,359, $-0{\cdot}047$ and $+0{\cdot}050$ in 4,078, and $+0{\cdot}059$ and $+0{\cdot}117$ in 4,047.

So far as we are aware, these relations as regards the positions and the intensities of the satellites are noticed for the first time.

It will be interesting to measure the intensity with different apparatus, as the results here obtained may have been affected by instrumental errors.

At the present stage we do not wish to advance any hypothesis explaining the distribution of intensity among the

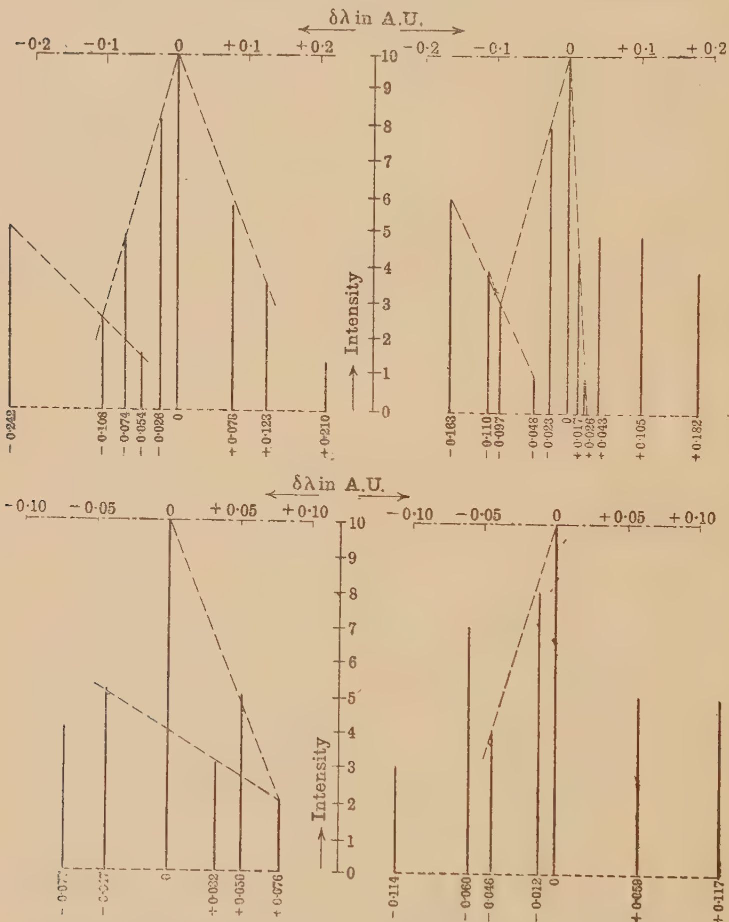

FIG. 36.—RELATIVE INTENSITIES OF THE SATELLITES OF THE LINES
λ=5,461. λ=4,359.
λ=4,078. λ=4,047.

satellites, but simply suggest that it will have important bearing in explaining the structure of complex spectrum lines, especially from the standpoint of the theory of ionic collisions.

It would also be interesting to see if these relations hold also for satellites of different lines of metals, as bismuth, cadmium, lead and others.

§ 11. *Conclusion.*—From the results of former observers, and from the present experiments, we see that the echelon spectrum is almost invariably accompanied by ghosts, and is ambiguous as regards the order of spectrum, when the satellites are not very near the principal.

These defects may be avoided, and the brightness of echelon spectrum combined with its high resolving power utilised, by crossing it with Lummer-Gehrcke plate, following the method initiated by Gehrcke and v. Baeyer.

The positions and intensities of the satellites of mercury lines are not entirely irregular, so that they seem to have some definite structure if properly arranged.

It is extremely desirable that similar researches should be extended to the spectrum lines of other heavy metals, and that the conditions under which the satellites appear or disappear should be fully investigated.

ABSTRACT.

The authors have photographed the principal lines of mercury, using an echelon spectroscope crossed by a Lummer-Gehrcke plate. They find that the 5,790 line consists of 8, the 5,769 line of 4, the 5,461 of 9, the 4,359 of 11, the 4,078 of 6 and the 4,047 of 7 components, whose positions in general agree with those found by recent observers. They point out a simple relation between the distances of the components from the principal line in each case, and a further relation between the quotient of each of these distances by the wave-length of the principal line, which holds for all the lines. The relative intensities of the component lines were determined by interposing an echelon photograph between a constant source of light and a linear thermopile, and noting the changes in the deflection of a galvanometer in series with the pile as the plate was moved across the face of the pile. In every case there appears to be a simple relation between the position and intensity of each component line.

DISCUSSION.

Prof. C. H. Lees pointed out that the ambiguity as to which order of spectrum any satellite belonged, could be very easily determined by transmitting the light from the echelon through a prism so as to increase or decrease the dispersion slightly, which would cause the satellites to open out from, or approach the principal line they belonged to proportionately, while the distance between the same line in adjacent orders would of course not be altered.

Prof. Stansfield was very much interested in the valuable work the authors had carried out, and objected only to their reflections on the character of the echelon spectroscope. He agreed with Prof. Lees that

the ambiguity as to the order of spectrum lines at some distance from the principal line, mentioned by the authors, could in practice be readily avoided by employing a prism to increase or decrease slightly the echelon dispersion. The echelon he had employed showed some of its secondary diffraction maxima, as any diffraction grating approaching to perfection in its optical behaviour was bound to do. These secondary maxima were abnormally bright on one side of the principal maximum and very faint on the other. In the Paper by Stansfield and Walmsley, referred to by the authors, this had been shown to be due to a cubic aberration produced by the one-sided clamping of the glass plates. In spite of this want of symmetry, however, he thought it was only fair to the instrument to call them secondary maxima, and he hoped that the authors of the Paper would say whether the faint lines they referred to as ghosts were also secondary maxima. His own list of components for the green line only differed from that given by von Baeyer and the authors of the present Paper in the omission of the faint line at $-54m$-Å., and this did not represent any difference of opinion as several of his photographs, including the one reproduced in his Paper, gave some evidence of this line. A secondary diffraction maximum which happened to come in that position was so bright that the presence of a faint primary assisting it was strongly suspected. Fabry and Perot's early values for the green line, which differed considerably from the others quoted in the Paper, were not regarded by their authors as accurate determinations. He was not aware that they had ever published them except in correspondence with Prof. Zeeman. He considered that the agreement between the results obtained by widely differing methods was fairly satisfactory. There was another effect which should not be overlooked when working with the echelon which was due to the light reflected forwards and backwards between the plates. This produced Fabry and Perot bands superposed on the ordinary spectrum. These were chiefly visible on broad lines, which thereby looked ribbed, and caused an erroneous splitting up of the lines. This could be detected by rotating the echelon slowly, when the Fabry and Perot bands will move fastest, and will, therefore, move over the surface of the line they are situated on.

Thanks were returned by the meeting to Prof. H. Nagaoka for having communicated his valuable Paper to the Society.

PLATE I.

Fig. 5.—Magnification: 8.

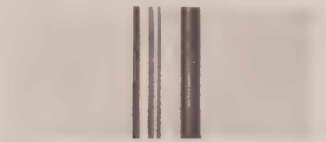

Fig. 6. Magnification: 7.

Fig. 7.—Magnification: 10.

To face page 30.]

PLATE II.

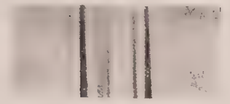

Fig. 9A.

Fig. 9B.—Magnification: 12.

Fig. 11.—Magnification: 12.

PLATE III.

Fig. 12.—Magnification: 7.

Fig. 14.—Magnification: 12.

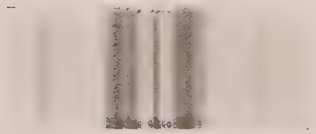

Fig. 16.—Magnification: 8.

PLATE IV.

Fig. 17.—Magnification: 6.

Fig. 18.—Magnification: 8.

Fig. 19.—Magnification: 10.

PLATE V.

Fig. 21.—Actual Size.

Fig. 22.—Magnification: 10.

PLATE VI.

Fig. 23.—Magnification: 6.

Fig. 25.—Magnification: 8.

Fig. 26.—Magnification: 8.

Fig. 27.—Magnification: 4.

To face page 30.]

PLATE VII.

Fig. 29.—Magnification: 10.

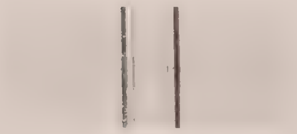

Fig. 30.—Magnification: 10.

PLATE VIII.

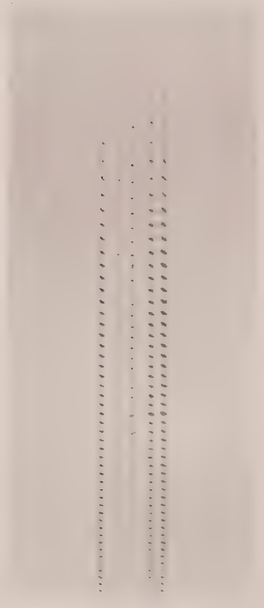

Fig. 32.—Magnification: 10.

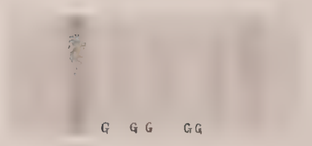

Fig. 33.—Magnification: 10.

PLATE IX.

Fig. 34.—Magnification: 8.

II. *Note on the Mutual Inductance of Two Coaxial Circular Currents.* By *Prof.* H. NAGAOKA, *Imperial University, Tokyo.*

RECEIVED AUGUST 27, 1912. READ OCTOBER 25, 1912.

THE " Bulletin " of the Bureau of Standards, Vol. VIII., No. 1 contains a résumé of different formulas for the mutual inductance of two coaxial circular currents. Several tables for the numerical evaluation of mutual inductance are given, but it appears to me that nearly all practical cases can be calculated with accuracy of about one part in a million, by means of a short table, which is applicable to all cases, for which the circles may be far apart or they may be *nearly* in contact.

Denoting the radii of the circles by A and a, and the distance between the centres by b, Maxwell* gave two formulas :

$$M=4\pi\sqrt{Aa}\left\{\left(\frac{2}{k}-k\right)K-\frac{2}{k}E\right\} \quad . \quad . \quad . \quad . \quad . \quad (1)$$

in which
$$k=\frac{2\sqrt{Aa}}{\sqrt{(A+a)^2+b^2}}=\sin\gamma=\frac{\sqrt{r_1^2-r_2^2}}{r_1},$$

and
$$M=8\pi\frac{\sqrt{Aa}}{\sqrt{k_1}}(K-E) \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (2)$$

in which
$$k_1=\frac{r_1-r_2}{r_1+r_2},$$
$$r_1=\sqrt{(A+a)^2+b^2},$$
$$r_2=\sqrt{(A-a)^2+b^2}.$$

K and E are complete elliptic integrals of the first and second kind respectively, with the corresponding modulus k or k_1.

From the general expression for the mutual inductance of two circles, or from Maxwell's first formula, we can deduce the following expression† in theta-functions :

$$M=-\frac{2\sqrt{Aa}}{\theta_2^2(o)}\left(\frac{\theta_0''(o)}{\theta_0(o)}+\frac{\theta_3''(o)}{\theta_3(o)}\right) . \quad . \quad . \quad . \quad . \quad (3)$$

On expansion, this becomes

$$M=4\pi\sqrt{Aa}\{4\pi q^{\frac{3}{2}}(1+3q^4-4q^6+9q^8-+\dots)\} . \quad . \quad . \quad (3')$$

* Maxwell, " Electricity and Magnetism," 2, § 701.
† Nagaoka, " Phil. Mag.", 6, p. 19, 1903.

The above series is rapidly convergent, and is convenient for numerical calculation, as the values of

$$\log\,(1+3q^4-4q^6+9q^8-+\ldots)$$

have been tabulated.* When k is near unity, we have to use q_1, which is complementary to q, and

$$M=4\pi\sqrt{Aa}\frac{1}{2(1-2q_1)^2}\left\{\text{log. nat.}\left(\frac{1}{q_1}\right)(1+8q_1-8q_1{}^2+\ldots)-4\right\}. \quad (4)$$

Although the above series converges very rapidly, it is rather tedious to find log. nat. $\left(\frac{1}{q_1}\right)$.

In Maxwell's second formula, we have the relation between k and k_1 given by

$$k=\frac{2\sqrt{k_1}}{1+k_1}.$$

From the theory of elliptic functions, we know that for the above transformation,

$$2\frac{K'(k')}{K(k)}=\frac{K'(k'_1)}{K(k_1)},$$

so that

$$q(k)=e^{-\pi\frac{K'(k')}{K(k)}}=e^{-\frac{\pi}{2}\cdot\frac{K'(k'_1)}{K(k_1)}},$$

$$=\sqrt{q(k_1)}$$

Denoting $q(k_1)$ by h, we have

$$h=q^2(k) \quad \ldots\ldots\ldots\ldots \quad (5)$$

By direct transformation, or from Maxwell's second formula, we get

$$M=4\pi\sqrt{Aa}\frac{\theta''_0(o)}{\theta'_1(o)}, \quad \ldots\ldots\ldots \quad (6)$$

which on expansion becomes

$$M=4\pi\sqrt{Aa}\left\{4\pi h^{\frac{3}{4}}\left(\frac{1-4h^3+9h^8-16h^{15}+25h^{24}-\ldots}{1-3h^2+5h^6-\ 7h^{12}+\ 9h^{20}-\ \ldots}\right)\right\}. \quad (6')$$

Since h is a small quantity, the fractional term is near unity; we may therefore write it $1+\varepsilon$, so that

$$M=4\pi\sqrt{Aa}\{4\pi h^{\frac{3}{4}}(1+\varepsilon)\} \quad \ldots\ldots\ldots \quad (7)$$

On expansion

$$\varepsilon=1+3h^2-4h^3+9h^4-12h^5+\ldots-$$

which is the same as that given in (3′), if we take account of the relation (5). Constructing the table of $\log_{10}(1+\varepsilon)$ for values of h from 0 to 0·2, we can easily calculate M by (7) for almost all

* Nagaoka, "Journ." Coll. Sci., Tokyo, 27, Art. 6, 1909; "Bull." Bur. Stand., 8, p. 215-220, 1912.

h	0	Δ	1	Δ	2	Δ	3	Δ	4	Δ	5	Δ	6	Δ	7	Δ	8	Δ	9	Δ
0·00	0·0000000	13	0·0000013	39	0·0000052	65	0·0000117	91	0·0000208	116	0·0000324	141	0·0000465	167	0·0000632	193	0·0000825	218	0·0001043	243
0·01	0·0001286	268	0·0001554	293	0·0001847	317	0·0002164	343	0·0002507	367	0·0002874	392	0·0003266	416	0·0003682	440	0·0004122	465	0·0004587	489
0·02	0·0005076	513	0·0005589	537	0·0006126	561	0·0006687	584	0·0007271	608	0·0007879	632	0·0008511	656	0·0009167	678	0·0009845	702	0·0010547	726
0·03	0·0011273	748	0·0012021	772	0·0012793	794	0·0013587	818	0·0014405	840	0·0015245	863	0·0016108	885	0·0016993	908	0·0017901	930	0·0018831	953
0·04	0·0019784	976	0·0020760	997	0·0021757	1019	0·0022776	1041	0·0023817	1063	0·0024880	1086	0·0025966	1106	0·0027072	1129	0·0028201	1150	0·0029351	1171
0·05	0·0030522	1194	0·0031716	1214	0·0032930	1235	0·0034165	1258	0·0035423	1278	0·0036701	1299	0·0038000	1319	0·0039319	1341	0·0040660	1362	0·0042022	1382
0·06	0·0043404	1403	0·0044807	1423	0·0046230	1444	0·0047674	1465	0·0049139	1485	0·0050624	1505	0·0052129	1525	0·0053654	1545	0·0055199	1566	0·0056765	1585
0·07	0·0058350	1605	0·0059955	1625	0·0061580	1645	0·0063225	1665	0·0064890	1684	0·0066574	1703	0·0068277	1724	0·0070001	1742	0·0071743	1762	0·0073505	1781
0·08	0·0075286	1801	0·0077087	1819	0·0078906	1838	0·0080744	1858	0·0082602	1876	0·0084478	1896	0·0086374	1914	0·0088288	1933	0·0090221	1951	0·0092172	1971
0·09	0·0094143	1989	0·0096132	2007	0·0098139	2025	0·0100164	2044	0·0102208	2062	0·0104270	2081	0·0106351	2098	0·0108449	2117	0·0110566	2135	0·0112701	2153
0·10	0·0114854	2171	0·0117025	2188	0·0119213	2207	0·0121420	2224	0·0123644	2242	0·0125886	2259	0·0128145	2278	0·0130423	2294	0·0132717	2312	0·0135029	2330
0·11	0·0137359	2347	0·0139706	2364	0·0142070	2381	0·0144451	2399	0·0146850	2415	0·0149265	2433	0·0151698	2450	0·0154148	2467	0·0156615	2484	0·0159099	2501
0·12	0·0161600	2517	0·0164117	2535	0·0166652	2551	0·0169203	2567	0·0171770	2584	0·0174354	2601	0·0176955	2618	0·0179573	2633	0·0182206	2651	0·0184857	2667
0·13	0·0187524	2683	0·0190207	2699	0·0192906	2716	0·0195622	2731	0·0198353	2748	0·0201101	2764	0·0203865	2780	0·0206645	2796	0·0209441	2812	0·0212253	2828
0·14	0·0215081	2844	0·0217925	2860	0·0220785	2875	0·0223660	2892	0·0226552	2906	0·0229458	2923	0·0232381	2938	0·0235319	2954	0·0238273	2970	0·0241243	2985
0·15	0·0244228	3000	0·0247228	3016	0·0250244	3031	0·0253275	3046	0·0256321	3062	0·0259383	3077	0·0262460	3092	0·0265552	3108	0·0268660	3123	0·0271783	3137
0·16	0·0274920	3153	0·0278073	3168	0·0281241	3183	0·0284424	3198	0·0287622	3213	0·0290835	3228	0·0294063	3242	0·0297305	3258	0·0300563	3272	0·0303835	3287
0·17	0·0307122	3302	0·0310424	3316	0·0313740	3332	0·0317072	3346	0·0320418	3360	0·0323778	3375	0·0327153	3390	0·0330543	3404	0·0333947	3418	0·0337365	3433
0·18	0·0340798	3447	0·0344245	3462	0·0347707	3476	0·0351183	3491	0·0354674	3505	0·0358179	3519	0·0361698	3533	0·0365231	3548	0·0368779	3562	0·0372341	3576
0·19	0·0375917	3590	0·0379507	3605	0·0383112	3618	0·0386730	3633	0·0390363	3646	0·0394009	3660	0·0397669	3675	0·0401344	3689	0·0405033	3702	0·0408735	3717
0·20	0·0412452		...		...		...		...		...		...		...		...		...	

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cases that occur in practice. Evidently the interval between $h=0$ and $h=0{\cdot}2$ corresponds to that between $\gamma=0°$ and $\gamma=89°\ 30'$ in Maxwell's table, which only extends from $\gamma=60°$ to $\gamma=90°$.

The accompanying table of $\log_{10}(1+\varepsilon)$ was calculated for me by Mr. T. Mishima, who made use of eight place tables of logarithms by Bauschinger and Peters.* The table can be used within a wide range, *i.e.*, when the circles are very widely apart till they are nearly in contact.

The following explanation for the use of the table will be perhaps necessary.

Calculate $$k'_1=\frac{2\sqrt{r_1r_2}}{r_1+r_2},$$

which follows from the relation $k'^2_1=1-k_1^2$, and find

$$l=\frac{1-\sqrt{k'_1}}{1+\sqrt{k'_1}}.$$

Then $$h=\frac{l}{2}+2\left(\frac{l}{2}\right)^5+15\left(\frac{l}{2}\right)^9+\ldots$$

Generally the first term is sufficiently accurate. Find $\log_{10}(1+\varepsilon)$, corresponding to the above value of h; then M will be given by (7). When great accuracy is required, it is advisable to calculate l by the formula

$$l=\frac{k_1^2}{(1+k'_1)(1+\sqrt{k'_1})^2}.$$

An example will suffice to show the use of the table.

Take $\mathrm{A}=a=25$ cm., $b=1$ cm.,

then $k_1=0{\cdot}9607920$, $\frac{l}{2}=0{\cdot}1550662$

$h=0{\cdot}1552463$

$\log(1+\varepsilon)=0{\cdot}0260142$ by the table.

$\mathrm{M}=1036{\cdot}666_3$ cm. by (7)

The last figure is, of course, uncertain, as the tables are of seven places. By using Legendre's table in Maxwell's formula (2), Rosa and Grover† give for the same case

$$\mathrm{M}=1036{\cdot}666_1 \text{ cm.},$$

* Bauschinger and Peters, "Logarithmisch-trigonometrische Tafeln mit acht Dezimalstellen," Leipzig, 1910.

† Rosa and Grover, "Bull." Bur. Stand., 8, p. 21, 1912.

which differs from the value found above by less than one part in a million.

If we have to use Maxwell's formula (1), and the table* compiled after it, we have to find the value of M for $\gamma=88^\circ\ 51'\ 14''$. It will be easily seen that the first and second differences are very large, so that the calculation is extremely tedious.

The chief advantage of the above table lies in the fact that nearly all cases in practice are included within a short range of argument h, and the calculation is easy to perform, as the *second* difference is generally small ($\leqq 26$).

ABSTRACT.

Methods are given for the rapid calculation of the mutual inductance of two coaxial circular currents. Maxwell's first formula is converted into theta-functions, and then expanded in a Jacobian q series. The logarithmic values of this series for various values of q have been tabulated in a previous Paper by the author. When the circles are near one another a series for M is given in terms of q_1, where q_1 is the complement of q. In this Paper the author treats Maxwell's second formula in a similar way. A table of the values of these series found, computed to six decimal figures by T. Tishima, is given. The chief advantages of this table are that nearly all practical cases are included within a short range of the argument, and the calculation is simple, as the numbers in the difference columns are small. By the help of these tables and series the mutual inductance between two coaxial currents can be easily computed to a high degree of accuracy.

DISCUSSION.

Mr. A. Campbell expressed great thanks for this communication. He had already used the tables given by Prof. Nagaoka in a former Paper on the subject published in Japan very largely, and those given in the present Paper covered still wider limits and enabled cases to be calculated which otherwise necessitated the use of Legrende's Tables of Elliptic Functions.

Dr. Russell expressed his interest in the Paper. He pointed out that the value of the mutual inductance was given to seven figures. It was certainly a step in advance to be able to evaluate so easily and to such a high degree of accuracy the simple expression for the mutual inductance. But in practice the wires used had appreciable thickness, and it was highly desirable that a more accurate formula be obtained so as to take this thickness into account.

Mr. F. E. Smith said he would like to express personal thanks for the Paper, and hoped that Prof. H. Nagaoka would also give tables for the mutual induction of a coil and a circle. The point raised by Dr. Russell of the thickness of the wires had been considered by Dr. G. F. C. Searle in a Paper on the Current Balance, who found that the effect was in general negligible. It was quite evident that the present Paper was both labour and time saving.

* Maxwell, "Electricity and Magnetism," 2, 701. Also, Appendix, "Bull." Bur. Stand. 8, p. 190-192, 1912.

III. *The Absorption of Gas in Vacuum Tubes.* *By* S. E. Hill, *B.Sc.*

Received August 30, 1912. Read October 25, 1912.

It has long been known that the continued discharge of a current through an ordinary glass vacuum tube causes a gradual diminution in the pressure. Plücker first noticed the effect while passing the discharge by means of a Ruhmkorff's coil through a Geissler tube.*

Later on Muller and De la Rue observed the change in pressure after passing a discharge for several minutes through similar Geissler tubes, and in their later experiments they had to give up the use of tubes sealed off from the pump.†

Hutchins‡ examined the spectra of the gas enclosed in a vacuum tube, and found that if a trace of any gas be admitted into an exhausted tube and the discharge be then passed through it, the spectrum of the gas gradually disappears, leaving only the spectrum of hydrogen. This latter spectrum arises probably from the water vapour condensed on the walls of the tube, and which is so difficult to remove. Since then the knowledge of this phenomenon has been used to obtain extremely low pressures in tubes. Thus a tube run continually will reach such a degree of exhaustion that the discharge is at last unable to pass. The method has also been used to obtain "hard" rays from Röntgen ray bulbs. Many investigations have been made into the causes of this disappearance of gas, but none of the experiments performed have allowed us to decide with certainty whether the action is a definite chemical action or merely a physical absorption. J. J. Thomson inclined to the latter theory; but the experiments of R. S. Willows,§ carried out at the same time, did not bear out this idea. Willows used as his source of current a battery, as lending itself better to quantitative results than the irregular current derived from a coil. His experiments were conducted with soda, lead and Jena glass, under varying conditions of surface, &c. He came to the conclusion that most, if not all, of the gas absorbed is to be accounted for by a chemical combination with the glass. Willows also found

* Plücker, "Pogg. Annal.," 1858, Vol. CIII., p. 91.
† Phil. "Trans.," Part I., 1878, p. 155.
‡ Hutchins, "Amer. Journ. of Science," Vol. VII., 1899, p. 61.
§ Willows, "Phil. Mag.," April, 1901.

that soda glass gave the greatest absorption, lead glass next and Jena glass least of all. He therefore recommended the use of Jena glass for vacuum tubes. Hydrogen was found to be absorbed by Jena and lead glass to a far less extent than air or nitrogen, and in the case of lead glass part of the hydrogen reappeared after resting the tube for some time.

In 1907 Campbell-Swinton* published his experiments on the subject. He found that after a tube had been made to absorb a large quantity of gas, and was then heated to its fusion point, bubbles of gas appeared in the glass, which came to the surface and there burst. These bubbles appeared to be in the walls at an average depth of 0·12 mm. The glass was next crushed to powder in a special vessel, and the spectrum of the absorbed gas at once became evident. His final conclusion was that the gradual disappearance was due to the mechanical driving of the gas into the glass, and not to any chemical combination.

Soddy and Mackenzie† repeated these experiments, and obtained the Campbell-Swinton effect of the bubbles in the glass, but came to the conclusion that the gas which causes these bubbles is not the discharge gas driven into the glass. The bubbles are, in all probability, a secondary effect due to the chemical decomposition of the glass under the influence of local heating produced during the bombardment. They suggest, also, that in glass there are always present sufficient undecomposed carbonates and sulphates to account for the effect. No conclusion was arrived at as to whether the absorption of the gas was chemical or mechanical. Various other suggestions have been made. Thus, it has been suggested that the gas is shot right through the walls of the tube, but one of Willow's experiments negatives this. It has been thought that the absorption takes place at the electrodes, but this interpretation takes no note of the fact that the effect is obtained with the electrodeless discharge.

When about to commence a fresh series of experiments it seemed probable that a study of the effect with the electrodeless discharge would lead to more definite results, and this for several reasons. In the first place, any possibility of electrode action is done away with, both as regards absorption

* Campbell-Swinton, "Proc." Roy. Soc., Series A, Vol. LXXIX., April, 1907.

† Soddy and Mackenzie, "Proc." Roy. Soc., Series A, Vol. LXXX., February, 1908.

and evolution of gas. This latter proves a continual source of difficulty throughout such experiments. Secondly, with the ring discharge, the ions are not directly shot into the glass, as is the case when metal electrodes are used. The possibility of this is still less if the bulb is screened to prevent any electrostatic effects. Under these conditions, that part of the disappearance of gas due to mechanical driving into the walls of the tube will be practically eliminated or, at least, greatly reduced. The following experiments were, therefore, carried out on lines suggested by Dr. Willows. The apparatus was arranged as in Fig. 1. A is the bulb under examination, having a volume of about 200 cc. It is connected, through drying tubes, to a Toepler pump and M'Leod gauge. The gas absorption was measured by the corresponding pressure change, the pump being cut off during the readings, in order to make this greater. The secondaries of a 6 in. induction coil were connected to a

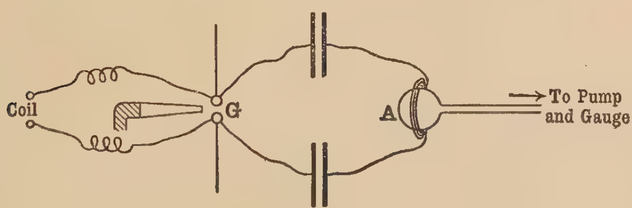

FIG. 1.

spark-gap, G, consisting of two zinc balls of diameter $1\frac{1}{2}$ in. The width of the spark-gap was maintained constant throughout the experiments at about 3 mm. Two Leyden jars were connected to the spark-gap, and from these went a coil of insulated wire, of about eight turns, which was placed round the bulb A. A great increase in the potential was obtained by blowing out the spark in the gap G. This was effected by means of a motor-driven blast. A discharge, almost invisible without the blast, became brightly luminous when the spark was blown out, and also much steadier.

The Leyden jars, spark-gap, &c., were insulated by standing on ebonite. The method of conducting an experiment was as follows :—

The gas was pumped down to a low pressure by means of a Fleuss pump. The exhaustion was continued by the Toepler pump, testing at intervals to see if the discharge could pass. As soon as this stage was reached the pressure was noted and

the spark-gap cleaned and adjusted. The discharge was then passed for a known time ($\frac{1}{4}$ hour) and the final pressure noted. The absorption could then be calculated, knowing the volume of the apparatus. The bulb was then exhausted farther and another reading taken. The series was continued until a stage was reached when the discharge would no longer pass. In no case could the discharge be made to pass at a higher

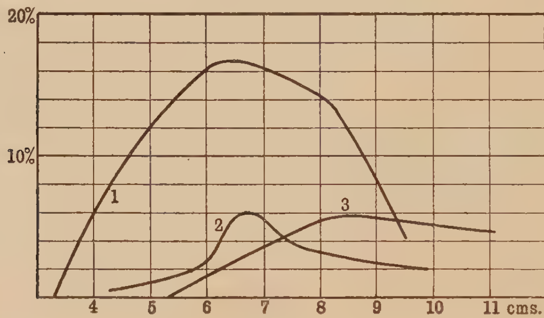

Fig. 2.—Soda Glass.

pressure than about 0·4 mm., or at a lower pressure than about 0·04 mm. After passing each discharge, the difference of pressure was calculated as a percentage of the mean pressure during the experiment.

The first bulb experimented on was of soda glass. A series of readings was taken as explained above and the results plotted on a curve. Fig. 2 shows the first curve obtained.

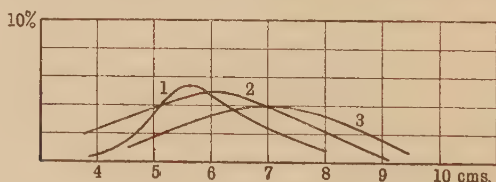

Fig. 3.—Bohemia Glass.

The ordinates are percentage absorptions, and the abscissæ the mean pressure during that absorption, given in centimetres of the gauge. The gauge read to $\frac{1}{500}$, therefore 10 cm. on the gauge represents $\frac{1}{50}$ of a centimetre actual pressure.

Some fresh air was then admitted to the bulb and allowed to stand over night to dry. Another series of readings was then taken, and after this a third series. The curves are marked

respectively 1, 2, 3. It will be seen that the absorption is less each time. The general shape of the curve remains the same in each case, but the whole is, as it were, shifted bodily along to the right, towards the region of higher pressures. The greatest absorption in the first experiment was 17 per cent. and in the last only 6 per cent. It seems, then, that the glass can be "tired" by continual working. The soda bulb was then replaced by one of Bohemia glass. The absorptions obtained with this were less than half those of the soda bulb. The

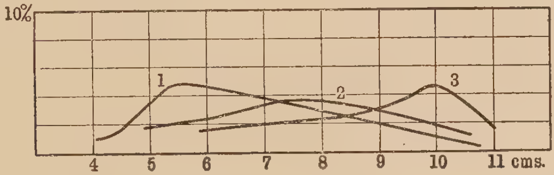

FIG. 4.—LEAD GLASS.

general shape of the curves (*see* Fig. 3) remains as before, the shift of the maximum towards the right again being noticeable.

A lead glass bulb was next tried. The general shape of the curves was again the same, the magnitude of the effect being about the same as with Bohemia glass (Fig. 4).

The last bulb used was of Jena glass. The absorption of air was of about the same magnitude as that of the lead and Bohemia bulbs (Fig. 5).

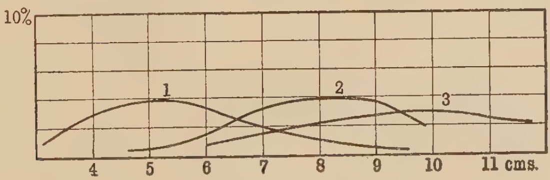

FIG. 5.—JENA GLASS.

It will be noticed that the "tiring" effect is not so great with the others as with the soda glass. The order of absorption was thus: soda, lead, Bohemia, Jena, decreasing from first to last.

The glasses were then laid aside for over two months, and the series was then repeated to see if they had recovered their power of absorption during this period. The results showed that the glass had recovered practically none of its absorptive power on standing. By the end of this second series of read-

ings the maximum absorption for the soda glass had decreased to $2\frac{1}{2}$ per cent., compared with an original 17 per cent. With the lead, Bohemia and Jena glasses similar results were obtained—*i.e.*, in each case the glass is becoming "saturated" with the gas. At this stage it was noticed that the soda and lead glasses had each a peculiar deposit on the neck of the bulb. The layer was of a dark brownish colour and its thickness very slight—of the magnitude of the wave-length of light, as shown by the colours it exhibited.

It was next decided to study the behaviour of hydrogen in the bulbs. If, during the above experiments, any chemical actions have taken place, it is natural to expect that various oxidation products of the glass have been formed. With hydrogen we should expect corresponding reducing actions. Assuming a chemical change to have taken place, we might expect a large initial absorption of hydrogen, going to reduce the oxidation products formed during the air experiments. On the other hand, if the gas is held in the glass mechanically, when an equilibrium state has been reached with air we should expect that a nearly steady state has been established for any other gas. On these grounds it ought to be possible to decide between the two theories that have been advanced to account for the absorption. The soda bulb was put on again and filled with hydrogen and pumped out several times. As soon as the discharge was started the brown layer on the neck of the bulb gradually began to disappear. The first absorption of hydrogen was enormous, the pressure falling from 14·45 cm. down to 5·05 cm. on the gauge, giving 95 per cent. absorption, calculated as above. This absorption then rapidly fell off, and after four series became much the same as that for air. The lead bulb was then filled with hydrogen, and with this the first absorption was 76 per cent. Here, again, the layer on the neck disappeared. With Bohemia glass the first absorption was 40 per cent., falling to 10 per cent. The Jena glass, which gave the smallest absorption with oxygen, gave the smallest with hydrogen, the maximum being about 8 per cent.

If, now, the hydrogen has reduced part of the oxidation products formed, it seems probable that it should have the effect of increasing the power of re-absorbing oxygen. To test this the bulbs were again filled with air, and its absorption noted. In all cases the absorption was found to have increased, although only with the Jena and Bohemia bulbs did it approach near to its original value. The curves are shown

in Figs. 6 and 7. In each case curve 1 is the last series with air before hydrogen was used, and curve 2 is the series with air after hydrogen was used.

In each case the hydrogen has had the effect of increasing the subsequent absorption of oxygen.

To see if any electrostatic effects of the discharge come into play, the bulb was wrapped round with wet blotting paper

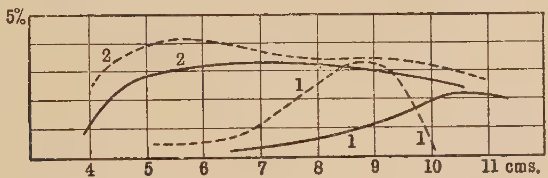

Fig. 6.
Full line Curves=Soda.
Broken line Curves=Lead.

and a series of absorptions taken. No difference in effects was obtained, however.

It was also noted that some of the bulbs showed the "after-glow" effect well. It was also thought that, if there had been any bombardment of the glass by the discharge, the phenomena of "thermo-luminescence," described by Wiedmann and Schmidt,* might be obtained as in the case of bombard-

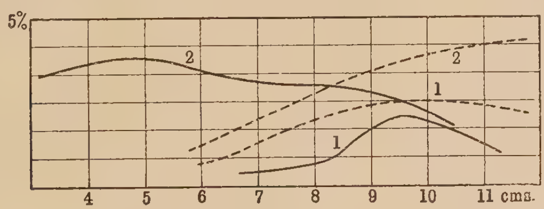

Fig. 7.
Full line Curves=Bohemia.
Broken line Curves=Jena.

ment by cathode rays. None of the bulbs, however, showed this effect. A review of the experiments shows that, in accordance with Willows' results, Jena glass is the best glass for vacuum tubes, as it shows least absorption. Even with soda glass the pressure variations can be reduced by saturating the glass with gas before it is sealed off from the pump. All that is required is to run the tube for some hours and admit

* Wiedemann and Schmidt, "Wied. Annal.," 59, 1895.

fresh gas until very little more is absorbed. Willows stated that Jena and lead glass absorb hydrogen to a less extent than air. This was found not to be the case, and his results are probably explained by the evolution of gas from his electrodes. The order of absorption with oxygen (air) gives soda, lead, Bohemia and Jena, and the same order holds for hydrogen.

To summarise : It seems most probable that the absorption is chiefly of a chemical nature. That some chemical changes are present is shown by the deposits on the necks of the bulbs. Thus, the deposit on the soda glass caused by air was removed by the hydrogen. A brown deposit on the Bohemia and lead glass likewise disappeared with hydrogen. With the Jena glass a slight colouration was given by oxygen, but with hydrogen a distinct black ring was formed which gradually diffused along the tube. Again, Strutt has shown that even nitrogen becomes unusually active in the discharge. These deposits, unfortunately, are too slight for chemical analysis, but they may be modifications of the alkalis in the glass. The absorption of the active gas hydrogen is much greater than that of oxygen, and the various glasses keep the same order of absorption with both gases.

Soddy and Mackenzie* obtained correspondingly small absorptions with the inert gases, and their experiments also tend to negative the mechanical theory of the absorption. It is, however, extremely difficult to devise a crucial experiment to definitely decide the cause of the absorption, and there is room for much more work on the subject.

In conclusion, I offer my best thanks to Dr. Willows for valuable guidance and suggestions during the experiments.

Cass Institute, E.C.

ABSTRACT.

This Paper is an account of experiments carried out to determine whether the absorption of gases caused by passing a discharge for some time through vacuum tubes is the result of a chemical action or is a mere physical absorption. In order to eliminate all electrode complications, the electrodeless discharge was used throughout. The bulbs examined were of soda, lead, Bohemia and Jena glass. The absorptions were noted at different pressures and curves plotted. Continued passage of a discharge causes a "saturation" effect in all the glasses. After two months none of the bulbs had recovered any of their absorptive power. If the action is chemical it is natural to expect various oxidation products to have been formed. Testing these bulbs with hydrogen we should expect a large initial absorption going to reduce these products. This was found to be the case for all

* Soddy and Mackenzie, *loc. cit.*

the bulbs, the first reading for the soda glass giving 95 per cent. absorption of hydrogen. Having now reduced the oxidation products we should expect a reabsorption of oxygen under the discharge. This was also found to be the case. The series of readings show great regularity, the order of absorption for oxygen holding also for hydrogen. That chemical actions are present is shown by peculiar deposits on the necks of the bulbs, these being unfortunately too small for analysis. The inert gases show correspondingly small absorption, as shown by Soddy and Mackenzie, and therefore the conclusion is that the disappearance is not due to physical absorption, but to definite chemical action.

DISCUSSION.

Dr. R. S. Willows mentioned the fact that the inert gases, like argon and helium, also disappeared on running the discharge, but this might be due to their being carried down by the cathode deposit. He had known a case of a hydrogen vacuum tube made of lead glass and provided with outside electrodes that was used as a detector for high-frequency oscillations, which failed to work after long continuous running, owing to the disappearance of the hydrogen, but worked again if the tube was left of one side for a week.

Mr. C. E. S. Phillips thought the electrodes played an important part in the diminution of pressure on working. Mr. Hill's results seem to indicate that the glass may become aged. It might be possible to age the glass for X-ray bulbs artificially. The constant change in hardness of X-rays owing to the change of pressure was a most serious drawback especially in medical work, where it was holding back the progress of the applications of X-rays to medicine.

Dr. G. W. C. Kaye remarked that in X-ray bulbs the spluttering of the cathode absorbed the gas. Campbell Swinton found some years ago that by heating the walls of a vacuum tube small bubbles of gas were given off, which was mostly hydrogen, and found that they came from distances up to 0·15 mm. below the surface. Ramsay and Collie had recently found helium to be evolved in a similar case. It was difficult to explain these results on the penetration theory, as the charged atoms would not penetrate the thinnest aluminium foil. This theory also does not explain the differences in behaviour of different kinds of glass. The results may be due to chemical activity excited in the gas by the discharge, such as has recently been found by Prof. Strutt to be the case with nitrogen. The violet coloration often noted in the glass of vacuum tubes was due to a sub-oxide of sodium. He suggested Mr. Hill experimented with silica bulbs.

Mr. A. A. Campbell Swinton contributed the following remarks: Mr. Hill alludes to my Paper published in the "Proceedings" of the Royal Society, Series A, Vol. LXXIX., 1907, but does not seem to be aware of my further Paper in the "Proceedings" of the Royal Society, Series A, Vol. LXXXI., 1908, in which the criticisms of Soddy and Mackenzie and of others are dealt with. In Mr. Soddy's experiments, as also in those of Mr. Hill, forms of electric discharge were employed with which the amount of heat communicated to the glass would be very great as compared with the amount of cathode-ray bombardment; whereas, in the arrangements adopted in my experiments, the converse was the case. Consequently, I do not think that the cause of the absorption of gas is necessarily at all the same in the one case as in the other, particularly as in my crucial experiment I used helium, which does not combine chemically with anything at ordinary temperatures.

The Author remarked that he would be glad to carry out the experiments suggested.

IV. *On a Method of Measuring the Thomson Effect.* By H. Redmayne Nettleton, *B.Sc., Assistant Lecturer in Physics at Birkbeck College.*

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1. Introduction.
2. Theory of Method.
3. Description of Apparatus.
4. Practice of Method.
5. Results of Experiment.
6. Discussion of Method and Results.

1. *Introduction.*

(*a*) When a current of electricity passes down a conductor in which a temperature gradient is maintained, the stationary value of the temperature at any place is dependent to a small extent on the direction of the current, for heat is evolved or absorbed in accordance with the Thomson effect.

Lord Kelvin expressed the relation between the heat evolved or absorbed, the temperature gradient and the current in the form $dQ = C\sigma \cdot d\theta$, where dQ is the heat evolved or absorbed per second by a current, C, in passing between two sections differing in temperature by $d\theta$. The quantity σ which may thus be measured in calories per 1°C. per ampere-second is called the specific heat of electricity.

Relative values of the Thomson effect in different metals have been obtained by Le Roux,* Trowbridge and Penrose,† and Battelli,‡ the last named experimenter indeed attempting to obtain an absolute value by measuring directly the heat produced by the effect in a given time in a definite portion of a bar under temperature gradient. It must be remembered, however, that the heat produced by the effect at once spreads down the conductor and influences the quantity of heat propagated by conductivity, time being required for a steady state to be obtained, and it can be shown that, besides the intrinsic difficulty of measuring directly such a small quantity of heat as that produced by the effect, Battelli's method does not fall

* Le Roux, "Ann. de Chimie et de Phys.," X., p. 258 (1867).

† Trowbridge and Penrose, "Phil. Mag.," 3 Ser., Vol. XIV., p. 440, 1882.

‡ Battelli, "Accad. delle Sci. di Torino, Atti," Vol. XXII., p. 518, 1887.

into harmony with the differential equation of distribution of temperature down a bar conveying an electric current—an equation first obtained by Verdet and given in his "Theorie Méchanique de la Chaleur." Haga* has devised a method, very general in its application, by which the specific heat of electricity can be obtained in absolute measure by comparing the change of temperature at a point when a current flowing down a conductor under temperature gradient is reversed with the rise in temperature produced by a current when the conductor is at uniform temperature throughout. Absolute measurements of the Thomson effect based on this method or on modifications of it have been made by Laws,† Lecher,‡ Schoute,§ Berg,|| and Aalderink,¶ while Callendar has devised a method based conjointly on Verdet's equation and the variation of electrical resistance with temperature which has been successfully carried out by King.**

The object of the present communication is to show: (*a*) That the modification of temperature gradient due to the heat produced or absorbed by the Thomson effect in a conductor of uniform cross-section passing through two constant temperature sources is exactly similar to the modification of gradient which would be produced by a definite slow uniform movement of the conductor itself in the proper direction.

(*b*) That for exact similarity of disturbance of gradient by impressed velocity and Thomson effect respectively—or for exact neutralisation—the following simple relationship holds:

$$\frac{\text{Current of electricity in amperes}}{\text{Material flow of conductor in grammes per second.}} = \frac{\text{Specific heat of electricity.}}{\text{Specific heat of material of conductor.}}$$

(*c*) That by working in the following manner the impressed velocity method may be applied to mercury to measure the absolute value of the specific heat of electricity, however large the Joule effect may be or whatever the emissivity loss:—

1. A current of C amperes is passed down a column of mercury heated at the top and maintained cold at the bottom. A

* Haga, "Ann. de l'École Polyt. de Delft," I., p. 145, 1885; III., p. 43 1886.

† Laws, "Phil. Mag.," Vol. VII., p. 560, 1904.

‡ Lecher, "Ann. d. Physik," XIX., p. 853-867, 1906.

§ Schoute, "Archives Néerlandaises," Série II., p. 175, 1907.

|| Berg, "Ann. d. Physik," XXXII.-XXXIII., pp. 477-519, June, 1910.

¶ Aalderink, "Archives Néerlandaises," Série II., p. 321, 1910.

** King, Amer. Acad., Proc., XXXIII., p. 353, 1898.

thermo-junction at a point near the middle of the gradient registers the temperature. On reversing the current the temperature is slightly raised, say, by $\Delta\theta_1$.

2. The current being still maintained (reversed) to keep the Joule effect and emissivity constant, a flow of mercury of, say, m grams per second is started up the tube, the temperature falling when the steady state is attained by $\Delta\theta_2$.

It can then be shown that :

$$\frac{2C\sigma}{ms}=\frac{\Delta\theta_1}{\Delta\theta_2},$$

and so σ is easily calculated.

The ratio of the two temperature changes can easily be made the ratio of two galvanometer deflections. By the use of two vertical columns in the form of an inverted U the sensitiveness is increased and the thermo-electric work is much simplified.

2. *Theory of the Method.*

Consider a vertical column of mercury flowing down a uniform glass-tube, heated at the top and cold at the bottom. The flow is very slow, so that the temperature changes produced

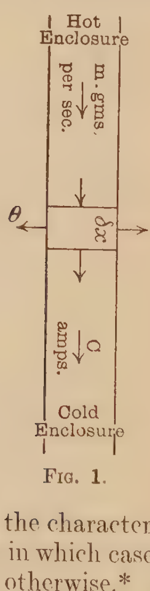


FIG. 1.

by it are very small, and the character of the isothermals is not appreciably altered by it, in which case it is immaterial whether the flow is parabolic or otherwise.* A current C amperes is also passing down the column.

*Nettleton, "Phil. Mag.," p. 590, April, 1910.

Let K=thermal conductivity of mercury.

s=specific heat of mercury.

σ=specific heat of electricity in mercury.

A=cross-section of the tube.

Let m=mass of mercury crossing a section of the tube per second.

The heat which enters per second a small prism fixed in space of depth δx and temperature θ is :—

(a) Due to conductivity $-\mathrm{KA}\dfrac{d\theta}{dx}$.

(b) Joule effect heat. If S=specific resistance of mercury, and α be the temperature coefficient of resistance, the resistance of the prism$=\mathrm{S}(1+\alpha\theta)\delta x/\mathrm{A}$; so if J is the number of joules equal to a calorie the heat produced in this way

$$=\frac{\mathrm{C^2S}}{\mathrm{J}}(1+\alpha\theta)\frac{\delta x}{\mathrm{A}}.$$

(c) Thomson effect heat$=-\mathrm{C}\sigma\dfrac{d\theta}{dx}\delta x$

(σ is assumed to be positive).

(d) Flow effect heat$=ms\theta$.

The heat leaving the prism per second is :—

(a)′ Due to conductivity $-\mathrm{KA}\dfrac{d}{dx}\left(\theta+\delta x\dfrac{d\theta}{dx}\right)$.

(b)′ Flow effect heat $=ms\left(\theta+\delta x\,.\,\dfrac{d\theta}{dx}\right)$.

(c)′ Due to emissivity $=\mathrm{E}p(\theta-\theta_0)\delta x$,

where θ_0=temperature of the enclosure,

p=perimeter of tube,

E=Newtonian coefficient of emissivity.

Equating the heat entering to the heat leaving, we obtain the differential equation pertaining to the steady state, which may conveniently be written—

$$\frac{d^2\theta}{dx^2}+a\frac{d\theta}{dx}+b\theta=c,$$

where
$$a=-\frac{(\mathrm{C}\sigma+ms)}{\mathrm{KA}},$$

$$b=\frac{1}{\mathrm{KA}}\left[\frac{\mathrm{C^2S}\alpha}{\mathrm{JA}}-\mathrm{E}p\right],$$

$$c=-\frac{1}{\mathrm{KA}}\left[\frac{\mathrm{C^2S}}{\mathrm{JA}}+\mathrm{E}p\theta_0\right].$$

If all temperatures be reckoned from the cold source as zero, θ_1 be the temperature of the hot source, and L the distance between the sources that is the entire length of the temperature-gradient, the temperature θ at a distance x from the higher source is given by the following general solution :—

$$\theta=\frac{c}{b}\left[\frac{e^{-\frac{ax}{2}}\sinh\lambda(x-\mathrm{L})-e^{-\frac{a(x-\mathrm{L})}{2}}\sinh\lambda x}{\sinh\lambda\mathrm{L}}\right]$$

$$-\frac{\theta_1 e^{-\frac{ax}{2}}\sinh\lambda(x-\mathrm{L})}{\sinh\lambda\mathrm{L}}+\frac{c}{b},$$

where $$\lambda=\frac{\sqrt{a^2-4b}}{2}.$$

If now we assume that the Thomson effect and flow effect are so small that the terms $\frac{ax}{2}$, $\frac{a\mathrm{L}}{2}$ are small compared with unity—and this condition is adhered to throughout the experiment—we may without making any approximation as to the magnitude of the "b" and "c" terms of the differential equation write the solution in the form,

$$\theta=a\mathrm{F}(\theta_1, x, \mathrm{L}, b, c)+f(\theta_1, x, \mathrm{L}, b, c),$$

or more shortly

$$\theta=a\mathrm{F}+f,$$

where F and f are functions of θ_1, x, L, b, c, but not of the "a" term in the differential equation.

This solution forms the basis of the method the values of F and f not being required as long as they are kept constant throughout. In general F and f are complex, but if it be assumed in addition that the emissivity loss is small and that consequently the term $\frac{b\mathrm{L}^2}{6}$ is small compared with unity a most useful solution may be obtained.

$$\theta=\theta_1\left(\frac{ax}{2}-1\right)\frac{x-\mathrm{L}}{\mathrm{L}}+\theta_1\left(\frac{ax}{2}-1\right)\left(\frac{x-\mathrm{L}}{\mathrm{L}}\right)\frac{bx(2\mathrm{L}-x)}{6}$$

$$+\frac{a}{2}(\mathrm{L}-2x)\tfrac{1}{6}cx(x-\mathrm{L})+\tfrac{1}{2}cx(x-\mathrm{L}).$$

This solution is most useful in showing the effect of the "b" and "c" terms, and in estimating the rise of temperature at a point due to reversing a current or due to superimposing a flow.

Case of Two Parallel Tubes.

Consider two parallel tubes, A and B, through which thermo-junctions are inserted near the middle of the respective gradients.

The temperature at " a " when a current is passing down the tube is given by $\theta = a\mathrm{F} + f$, and at " B " likewise when a current or flow is passing down by $\theta' = a'\mathrm{F}' + f'$. When a current is

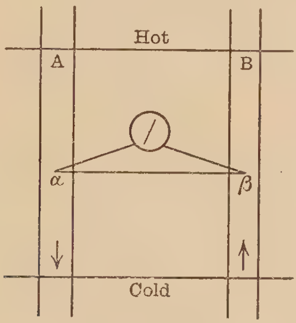


FIG. 2.

passing down A it will be passing up B, and hence if μ is the ratio of the cross-section of A to the cross-section of B, for all values of current and flow we have

$$a' = -\mu a.$$

We may now easily express the difference of temperature between " α " and " β " for the following three cases, making use of the suffixes c, v to indicate respectively an electric current or flow of mercury down A and hence up B.

of Electric nt or Flow.	Temperature of α.	Temperature of β.	Difference of Temperature between α and β.
(1) ent down nd up to B	$\theta_c \quad = a_c\mathrm{F} + f$	$\theta'_{-c} \quad = -\mu a_c\mathrm{F}' + f'$	$\Delta\theta_c \quad = a_c(\mathrm{F} + \mu\mathrm{F}') + f - f'$
(2) ent up A down B	$\theta_{-c} = -a_c\mathrm{F} + f$	$\theta'_c \quad = \mu a_c\mathrm{F}' + f'$	$\Delta\theta_{-c} \quad = -a_c(\mathrm{F} + \mu\mathrm{F}') + f - f'$
(3) ent up A down B; w of mercury down A up B	$\theta_{v-c} = (a_v - a_c)\mathrm{F} + f$	$\theta'_{-v+c} = \mu(-a_v + a_c)\mathrm{F}' + f'$	$\Delta\theta_v{}^{-c} = (a_v - a_c)(\mathrm{F} + \mu\mathrm{F}') + f - f'$

Now, $\Delta\theta_c - \Delta\theta_{-c}$ is measured directly by the deflection of the galvanometer spot on the scale; call this deflection d; it is the deflection due to reversing the current.

Similarly, $\Delta\theta_v^{-c} - \Delta\theta_{-c}$ is the deflection caused by superimposing a flow on the reversed current; call this deflection d_v.

Then $$d_c = 2a_c(\mathrm{F} + \mu\mathrm{F}')$$

and $$d_v = a_v(\mathrm{F} + \mu\mathrm{F}'),$$

and hence $$\frac{d_c}{d_v} = \frac{2a_c}{a_v} = \frac{2\mathrm{C}\sigma}{ms}$$

or $$\sigma = \frac{s}{2\mathrm{C}} \times \frac{m}{d_v} \times d_c.$$

In each experiment several values of d_c and of m/d_v are found, and the mean of each set used in the calculation for σ.

3. *Description of Apparatus.*

The apparatus, which has been greatly modified during the progress of the research, will be seen on reference to Fig. 3 to consist essentially of two parallel glass experimental tubes T_1, T_2, clamped at their lower ends within the large zinc vessel V, and passing above into an annular heater, A, near the top of which they are joined by an inverted U piece. The left-hand tube T_1 is joined below by wide pressure tubing to the glass head H, while the right-hand tube T_2 is similarly connected to the glass bulb tube B, and thence by narrow pressure tubing to the system of capillaries, which form the flow resistance. The mercury, which fills the apparatus, may be made to convey an electric current, which enters by the head H, and leaves the bulb tube B by eight platinum wires fused through the glass and dipping below into the mercury reservoir D. By releasing the pinch cock F, the mercury may be caused to flow slowly from the head H, through the apparatus and the attached capillaries. When the vessel V is full of cold water, and steam is circulating through the annular space of the heater A, a temperature gradient, about 8 cms. long, is obtained in the middle of the experimental tubes T_1 and T_2, temperature changes near the middle of each gradient being measured by the thermo-junctions T_{J1}, T_{J2}, of iron and constantan sealed through the glass tubes.

The following is the method of preparing and mounting the experimental tubes, great care being taken in the insertion

and insulation of the thermo-junctions. A cane of glass, as uniform in cross-section as possible, the internal diameter being just under 7 mm. and the external diameter about 1 cm., was divided into four parts. Two of the parts were prepared, as shown in Fig. 4, by Messrs. A. C. Cossor, Ltd., the iron

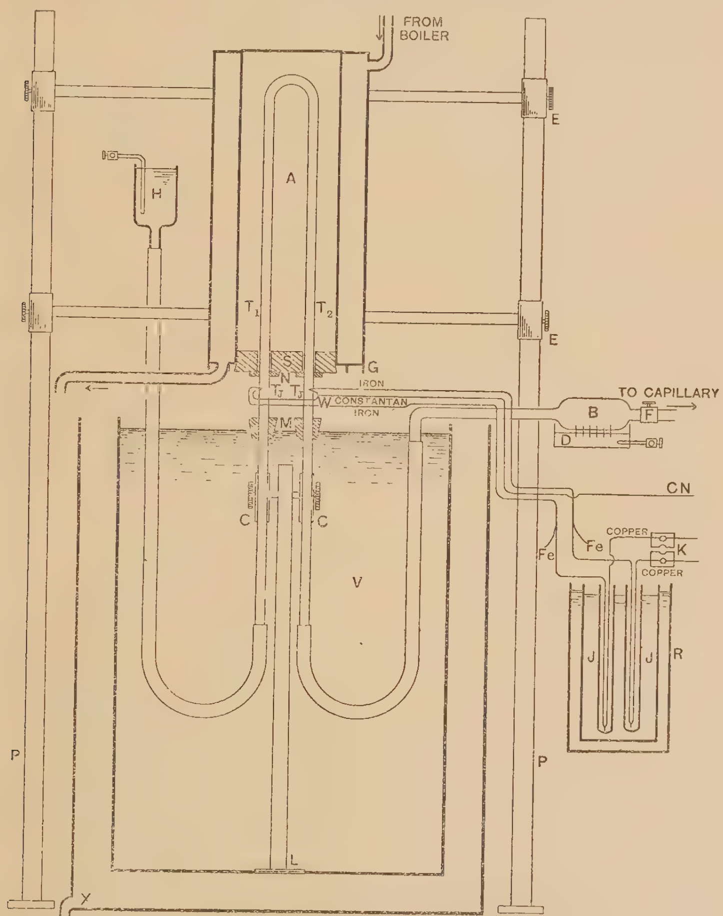


FIG. 3.

and constantan wire being passed through glass beads, the junction formed, and the insertion effected with a minimum upset of internal cross-section. The wires were in the same vertical plane, the junction being about 3·5 cms. from the nearer end of the tube and some 25 cms. from the further end.

Several such tubes have at different periods been supplied to the author, and the glass work brought to such perfection that in the tubes used in the present apparatus internal distortion was hardly visible. The next procedure was the insulation of the wires within the tubes, which was effected first with a thin layer of black club enamel, which flows very easily, and then, after drying with hot air, with a coating of red household hard enamel, use being made of a very fine camel-hair brush. Practice was required before neat insulation could be carried out and rough or unsuccessful coatings of enamel could easily be dissolved away by ether. The tubes, after thorough drying,

Fig. 4.

were returned to the glass-blower, who skilfully joined on the top portions with inappreciable disturbance and without affecting the insulation. The pressure tubing and corks M_1, n_2, M_2, n_2 were then put on the experimental tubes, which were also passed through the loose holes of the large cork S. Both tubes being held vertically, they were adjusted relatively to one another until the thermo-junctions were as nearly as possible in the same horizontal line : thus fixed by the double clamp C they were returned to the glassblower to be joined together by a U piece.

The apparatus was now filled with mercury, which had been distilled and purified by falling in fine drops through dilute nitric acid, and the clamp holding the tubes was then itself clamped at a convenient height on the firm iron upright fixed

within the zinc vessel V. The annular heater was now lowered down its supporting pillars P_1, P_2, towards the cork S, which could easily be made to fit it by adjusting the position of the outer vessel X and its contents. The heater and cork "S" were then lowered to the final position, the corks n_1, n_2 being brought up against the cork S, and closing the space above to air currents. Cotton wool was carefully wound round the tubes between the corks as well as round the annular heater, underneath which were soldered concentric strips of gauze forming a hold for cotton wool, which could be packed into the spaces lying between them and the outside of the cork S. The vessel V was filled with cold water up to the middle of the corks M_1, M_2, which had previously been boiled in paraffin wax. By covering the water with a layer of Fleuss-pump oil its evaporation into the cotton wool above was prevented.

The two constantan wires were joined together at W, and the two iron wires lead away to the copper leads shown in the diagram, the junctions of copper and iron being side by side in two glass tubes J, J, immersed in water in the Newton annular cooling calorimeter R. The ends Cn, Fe_1, Fe_2 of the wires were of use only when the actual temperature of the junction T_{J1}, T_{J2} were required. The iron constantan couple has not only the merit of being suitable for fusing through glass, but also possesses a remarkably high thermo-electric power—some 50 microvolts per degree centigrade—the relation between electromotive force and temperature being for the range 0°—100°C. strictly linear.

The galvanometer was a quartz fibre mirror instrument of the Broca type, the equivalent resistance of the two coils in parallel being 8·28 ohms, and the sensitiveness such that a micro-ampere produced about 11 cms. deflection on the scale distant nearly 2 metres from the mirror. The resistance of the rest of the circuit was 1·18 ohms, and a degree alteration in difference of temperature between the thermo-junctions was represented by 60 cms. deflection. There was no movement of the galvanometer mirror on quickly reversing a current of 8 amperes whether the galvanometer circuit was open, closed, or short-circuited, thus showing both that the instrument was sufficiently far away from the experimental tubes to render magnetic disturbance negligible, and that the insulation of the thermo-junctions was satisfactory.

An operation of importance is the adjustment, when steam is passing through the heater of the thermo-junctions T_{J1}, T_{J2},

to the same temperature, so that when the galvanometer circuit is closed the line of light is suitably situated on the scale. It was anticipated that this would be troublesome, and earlier apparatus was so designed that each experimental tube could be raised or lowered relatively to the other. Experience showed, however, that this was absolutely unnecessary, for if the junctions were set by eye to be as nearly as possible in the same horizontal line, the fine adjustment could easily be made by reducing the emissivity loss on one tube or the other by the addition of a small piece or two of cotton wool, inserted with the aid of a knife or prong, to that already round the tubes between the corks. This method of adjustment is not only convenient, but also brings out most clearly the need for complete firmness and rest of the wool during an experiment, emphasising the necessity for protection from draughts, convection eddies and vibration.

The currents used during the research varied from 4 to 9 amperes, and were derived from the 100-volt mains in series with which were a woven-wire resistance with six steps, a 0–20 ohm rheostat in 40 steps, a standard $\frac{1}{100}$ ohm resistance, a hot-wire ammeter, and the reversing commutator and main apparatus. In multiple arc with the woven-wire resistance were a water resistance and 0–1 Weston ammeter, the water resistance in which the distance between the zinc electrodes could be varied forming a most suitable fine adjustment for the current in the main circuit. During the night the current from the mains was much steadier than during the day, but prior to any reading the current was for some time closely watched and regulated if necessary by the water rheostat.

It remains to describe the method of obtaining a constant flow, which will cause temperature changes of the same order as those produced by reversing a current of 5 or 6 amperes. Calculation shows that a flow of 1 gram per hour is about the equivalent of reversing 3 amperes, while before it was decided to proceed with the method, preliminary experiments made two years ago showed that flows of mercury of about 2 grams per hour could be obtained constant to 1 per cent. without any special precautions. Most of the flows used in the apparatus described were as large as 3·5 grams per hour and, at least, constant to one part in 150, but, what is more important, the ratios mean flow to galvanometer deflection produced by the flow were at least constant to 2 per cent., which is as high an accuracy as can be obtained in the value of the galvanometer

deflections produced by current reversals. It is interesting to note that an outflow of 5 grams per hour, which is faster than any used in the research, would roughtly mean a linear velocity through the experimental tubes of about 1 cm. per hour. The flows then being very small, it is unnecessary to maintain a constant head by any overflow arrangement, but the head tube H was allowed a width of about 5 cms. The resistance consisted of about 2 metres of thick-walled thermometer tubing of fine bore, on to which was joined by pressure tubing a piece of ordinary glass tubing drawn out into a very fine capillary over a metre long. The last and finest portion of this capillary lay on a wooden inclined plane, by altering the inclination of which the value of the flow could be modified. The drop period varied from about 4 to 8 seconds, according to the magnitude of the flow which was measured, as described in the next section.

4. *Practice of the Method.*

An experiment was usually started between three and four o'clock in the afternoon by preparing the boiler and starting the flow of steam through the annular heater. The current was started after steam had been passing for an hour, and after another hour the short-circuit plug of the galvanometer key was removed and the position of the line of light on the galvanometer scale examined and modified, if need be, by the addition of a little piece of cotton wool to one side or other of the surroundings of the experimental tubes. Even after the attainment of the steady state the galvanometer needle still showed signs of irregular movement, but towards 10 at night the deflection became much steadier, and it was possible to proceed with the experiment. This "zero movement" has caused the author, and many other experimenters on the Thomson effect, the greatest trouble; in earlier apparatus used in the research it was large and progressive, and was attributed in part to the warping of wood which supported the annular heater and glass tubes. In the present apparatus the supports were all very stable; convection currents from the heater were allayed by the free use of cotton wool, and humidity changes within the wool were prevented by covering the water in the cooling tank with a layer of Fleuss pump oil. So great was the improvement brought about by the modification of the apparatus and by working at night that often, though not always, the needle would behave almost perfectly, good behaviour being illus-

trated by the reversal readings of Experiment IV., given in the next section, and relatively bad behaviour by the reversals of Experiment VI.

The deflection having become steady, the statical part of the experiment or " the reversals " was proceeded with as follows : The needle of the ammeter in the main circuit was maintained by the water resistance in one of the branch circuits accurately on the current reading chosen, which was always an exact number of amperes, for some 10 or 15 minutes prior to taking the galvanometer deflection. When this had been read the commutator was reversed and the new steady state waited for—a period of about 35 minutes, during the last 10 of which the movement was very little. A uniform period of 40 minutes was allowed between each reversal, the current being closely regulated for 10 minutes before each reading. The deflections due to reversing the current were then found by subtracting the mean of two consecutive readings with the current in the same direction from the intermediate reading with the current in the opposite direction.

Sufficient reversal readings having been obtained, and the current being in the direction causing the line of light to lie on the right-hand side of the scale, the dynamical or flow part of the experiment was proceeded with by observing the deflection as before, short-circuiting the galvanometer, and releasing the pinch-cock whereby a flow of mercury through the apparatus could be started. The release of the compressed pressure-tubing evidently caused a momentary outrush of mercury, for, unless the galvanometer was short-circuited, the line of light was thrown off the scale towards the left. For this same reason, too, the velocity of the mercury was not measured until the flow had settled down.

The method of obtaining the flow readings and flow magnitude is best illustrated by an example :—In Experiment I. the last reversal reading was at 1·0 a.m., when the deflection was 9·75 to the right. The galvanometer was at once short-circuited and the pinch-cock released very small drops of mercury commencing to fall from the end of the fine capillary. By 1·15 it was safe to re-open the galvanometer circuit, the line of light being on the scale, but well to the left. At 1·25 the measurement of flow was commenced, a weighed watch-glass being placed under the end of the capillary as soon as the first drop after 1·25 had fallen. Meanwhile the deflection had approached a steady value, which, after 10 minutes watching

of the current, was read immediately after 1·42 as 19·7 to the left; the galvanometer was then short-circuited. The watch-glass was then carefully removed the moment the first drop after 1·45 had fallen, and the pinch-cock was then at once screwed up tightly. After 2 o'clock the galvanometer circuit was opened, and the final reading taken at 2·25, after steadying the current, was 10·0 to the right. The intervals between galvanometer readings were thus practically equal, and the mean of 9·75 and 10·0 added to 19·7 gave the flow deflection 29·58 corrected for zero change. This was now divided into 1·0674, the number of grammes of mercury outflowing in the 20 minutes, the values of M/*d* in the tables below thus being obtained. The above process was repeated, usually three times, using flows of different value, the modification of flow being easily effected by raising or lowering the inclined plane on which the end portion of the fine capillary rested.

That different values of M/*d* should be obtained on different days is only to be expected from the theory which shows that it will depend slightly on the current and emissivity alterations, but more largely and directly on the difference of temperature between the hot and cold sources. Similarly, in the statical experiment the ratio C/*d* is only theoretically and practically an approximate constant. Any change, however, which affects the ratio M/*d* will equally affect the ratio C/*d*, so that changes of temperature in the room would in no way impair the accuracy of the values obtained, provided the second part of the experiment follows on immediately after the first.

At the conclusion of the flow portion of the experiment the actual temperature of each thermo-junction was found by balancing its electromotive force against that of a similar junction tied to a good standardised thermometer immersed in hot water in a long cylindrical vacuum vessel.

The actual current values taken were not based on the readings of the hot-wire ammeter but on the electromotive force across the standard $\frac{1}{100}$ ohm resistance as measured on a Paul millivoltmeter, which could not always be spared from the laboratory until late in the evening. This millivoltmeter was most carefully calibrated, both before and after the series of six experiemnts, on a Crompton potentiometer against a new Weston cell, which agreed to one part in 400 with an older Clark cell.

The only other value required before calculating the value of σ is s the specific heat of mercury. This is taken as 0·03294,

which is Barnes* value at 62°C. and which agrees very closely with the value given by Winkelmann.

5. *Results of Experiments.*

The first table given is a verification of the equation $\theta = aF + f$ for flows, and shows that they are small enough to permit of the one approximation made in the theory of the method.

Hour.	Galvanometer readings.		Mean deflection d.	Mass outflowing in 20 min. M.	M/d.
9·55		19·55			
10·37	22·2		40·58	1·4885	0·0367
11·20		17·2			
12·2	21·5		38·70	1·4194	0·0367
12·45		17·2			
1·27	15·0		32·43	1·2024	0·0371
2·10		17·65			
2·52	6·3		24·15	0·8884	0·0368
3·35		18·05			
4·17	6·45		24·78	0·8984	0·0363
5·0		18·60			
5·42	2·65		21·75	0·7978	0·0367
6·25		19·60			
7·7	9·45		28·88	1·0682	0·0370
7·50		19·25			

In this experiment, which proves that the flow of mercury is proportional to the temperature change it produces, no electric current was running.

The results of six consecutive experiments using differing currents are now given :—

Experiment I.

Date June 12–13, 1912.
Temperature of thermo-junctions, 60·2°C.
Magnitude of current, 6·24 amperes.

TABLE A.—*Reversals.*

Hour.	Galvanometer readings.		Deflection d_c due to reversal of current.
7·0	9·3		
7·40		10·25	19·20
8·20	8·6		18·93
9·0		10·4	19·30
9·40	9·2		19·35
10·20		9·9	19·08
11·0	9·15		19·10
11·40		10·0	19·15
12·20	9·15		19·03
1·0		9·75	

Mean value of $d_c = 19{\cdot}14$.

* Barnes, Brit. Assoc. Report, p. 530, 1902.

TABLE B.—*Flows.*

Hour.	Galvanometer readings.		Deflection.	Mass M outflowing in 20 mins.	M/*d*.
1·0		9·75			
1·42	19·7		29·58	1·0674	0·0361
2·25		10·0			
3·7	9·4		20·20	0·7248	0·0359
3·50		11·6			
4·32	21·4		32·8	1·1552	0·0352
5·15		11·2			
5·57	19·85		30·95	1·1080	0·0358
6·40		11·0			

Mean value of $M/d=0·0357[5$.

whence $$\sigma=\frac{0·03294}{2}\times\frac{19·14}{6·24}\times\frac{0·03575}{1200}.$$
$$=0·00000151.$$

Experiment II.

Date June 14th and 15th.
Temperature of thermo-junctions, 61·4°C.
Magnitude of current, 7·24 amperes.

TABLE A.—*Reversals.*

Hour.	Galvanometer readings.		Deflection d_c due to reversal of current.
11·15		16·1	
11·55	6·75		22·70
12·35		15·8	22·43
1·15	6·50		22·40
1·55		16·0	22·50
2·35	6·50		22·68
3·15		16·35	

Mean value of $d_c=22·54$.

TABLE B.—*Flows.*

Hour.	Galvanometer readings.		Deflection.	Mass M outflowing in 20 mins.	M/*d*.
	Height of capillary tube altered by accident.		Not taken.	Not weighed.	...
4·40		17·45			
5·22	13·0		30·58	1·0707	0·0350
6·5		17·70			
6·47	13·25		31·18	1·1005	0·0353
7·30		18·15			
8·12	12·10		30·25	1·0810	0·0357
8·55		18·15			
9·37	12·85		30·93	1·0855	0·0351
10·20		18·0			

Mean value $M/d=0·0353$.

whence $\sigma=0·00000151$.

Experiment III.

Date June 18th and 19th.
Temperature of thermo-junctions, 63·7°C.
Magnitude of current, 8·25 amperes.

Table A.—*Reversals.*

Hour.	Galvanometer readings.		Deflection d'_c due to reversal of current.
10·10	10·1		
10·50		16·2	26·13
11·30	9·75		25·85
12·10		16·0	25·75
12·50	9·75		25·80
1·30		16·1	26·00
2·10	10·05		26·10
2·50		16·0	

Mean value of d_c=25·94.

Table B.—*Flows.*

Hour.	Galvanometer readings.		Deflection.	Mass M outflowing in 20 mins.	M/*d*.
4·15		17·5			
4·47	19·15		36·6	1·3044	0·0356
5·40		17·4			
6·22	12·85		30·6	1·0908	0·0356
7·5		18·1			
7·47	13·9		31·8	1·1392	0·0358
8·30		17·7			
9·12	14·4		32·4	1·1703	0·0361
9·55		18·3			

Mean value of M/*d*=0·0358.

whence σ=0·00000155.

Experiment IV.

Date June 21st and 22nd.
Temperature of thermo-junctions, 61·3°C.
Current, 5·12 amperes.

Table A.—*Reversals.*

Hour.	Galvanometer readings.		Deflection d_c due to reversal of current.
10·0		16·65	
10·40	1·05		15·60
11·20		16·65	15·55
12·0	0·95		15·60
12·40		16·45	15·50
1·20	0·95		15·60
2·0		16·65	

Mean value of d_c=15·57.

TABLE B.—*Flows.*

Hour.	Galvanometer readings.		Deflection.	Mass M outflowing in 20 mins.	M/*d*.
2·0		16·65			
2·42	5·3		22·25	0·8091	0·0364
3·25		17·25			
4·7	6·0		23·53	0·8586	0·0365
4·50		17·8			
5·32	13·95		31·9	1·1653	0·0365
6·15		18·1			
6·57	14·65		32·68	1·2055	0·0369
7·40		17·95			

Mean value of M/d=0.0366.

whence σ=0·00000153.

Experiment V.

Date, June 26th and 27th.
Temperature of thermo-junctions, 60·6.
Magnitude of current, 4·10 amperes.

TABLE A.—*Reversals.*

Hour.	Galvanometer readings.		Deflection d_c due to reversal of current.
11·10		17·85	
11·50	5·45		12·28
12·30		17·6	12·22
1·10	5·30		12·30
1·50		17·6	12·20
2·30	5·5		12·33
3·10		18·05	

Mean value of d_c=12.266.

TABLE B.—*Flows.*

Hour.	Galvanometer readings.		Deflection.	Mass M outflowing in 20 mins.	M/*d*.
3·10		18·05			
3·52	9·2		27·98	1·0351	0·0370
4·35		19·5			
5·17	21·3		40·44	1·5128	0·0374
6·0		18·78			
6·45	18·2		37·14	1·3953	0·0376
7·25		19·1			
8·8	19·65		33·45	1·4435	0·0375
8·50		18·5			

Mean value of M/d=0.0374.

whence σ=0·00000154.

Experiment VI.

Date, June 28th and 29th.
Temperature of thermo-junction, 67·5°C.
Magnitude of current, 9·26 amperes.

TABLE A.—*Reversals.*

Hour.	Galvanometer readings.		Deflection d_c due to reversal of current.
9·10	9·4		
9·50		19·0	28·95
10·30	10·5		29·32
11·10		18·63	29·16
11·50	10·55		29·44
12·30		19·15	29·38
1·10	9·90		29·48
1·50		20·0	29·33
2·30	8·75		28·95
3·10		20·4	

Mean value of d_c=29·25.

The zero movement was worse than usual at night.

TABLE B.—*Flows.*

Hour.	Galvanometer readings.		Deflection.	Mass M outflowing in 20 mins.	M/*d*.
3·10		20·4			
3·52	13·45		34·85	1·2815	0·03677
4·35		22·4			
5·17	16·95		39·29	1·4426	0·03672
6·0		22·28			
6·42	18·58		41·32	1·5220	0·03683
7·25		23·2			
8·7	16·0		38·98	1·4295	0·03667
8·50		22·75			

Mean value of M/*d*=0.03675.

The flows used were all large and more constant than usual.
This experiment gives σ=0·00000159.

The results are summarised below :—

Current in amperes.	Temperature.	Value of σ is calories per degree Centigrade per coulomb.
4·1	60·6	0·00000154
5·12	61·3	0·00000153
6·24	60·2	0·00000151
7·24	61·4	0·00000151
8·25	63·7	0·00000155
9·26	67·5	0·00000159

On ascertaining the direction of the current the Thomson effect was seen to be negative for a flow of mercury down one

of the experimental tubes from hot to cold had a similar effect to reversing a current initially passing downwards.

6. *Discussion of Method and Results.*

While the theory of the method is general, it would seem limited in practice to mercury and amalgams. Again it is more important than in Haga's method that adequate time be allowed for the attainment of the steady state; for if the time were insufficient the deflections due to current reversal would be too small, while the deflections due to impressed velocity would be larger than their ultimate value and so too low a value of the effect would be obtained. Probably this objection would disappear if the flows were started by slowly opening a tap.

On the other hand, the flow effect and Thomson effect are similar terms in the equation of the method, their influence on the emissivity loss being similar and negligible; they can, moreover, be made to give deflections of the same order; on the contrary, the Thomson effect and Joule effect are dissimilar and dependent, and in Haga's method in which a comparison between these two effects is made a measurement of temperature gradient is necessary, and deflections of the same order can only be obtained at the expense of differences in emissivity conditions. In practice the flow portion of the experiment has never presented the slightest difficulty, but much trouble has been taken in attaining suitable deflections on reversing the current and, above all, in securing consistency in their value by reducing to a minimum variations of zero.

The author believes that the method will be very suitable for finding the variation of Thomson effect with temperature, but has not attempted it with the present apparatus in which only the higher temperature source could conveniently be raised. Schoute's results indicate that the variation with temperature is large, and it is thus not advisable to determine the temperature coefficient by increasing the temperature-gap. Moreover, it is desirable to shorten the time needed for attainment of the steady state.

The results obtained by the author are in fair agreement with those obtained by Schoute, employing a modification of Haga's method. Schoute obtained values, somewhat inconsistent, varying from as low as 0·00000134 at 32°C. to as high as 0·00000248 at 154°C. the value changing from 0·00000160 at 53°C. to 0·00000180 at 58°C. The only other experimenter on

the effect in mercury is Haga, who, working as far back as 1885, obtained a value as low as 0·00000069 at 78°C., but owing to the difficulties under which he worked his results are very inconsistent; moreover, the Joule effect and Thomson effect temperature changes were measured under different conditions and in different ways, and what is serious in this particular method a very large upset of cross-section was caused by the introduction of his thermo-junctions.

In conclusion, the author must express his thanks to Principal Armitage-Smith, for facilities for carrying out the work, all of which was done through the night; to Dr. A. Griffiths, head of the Physics Department, for his great interest, and to Mr. J. L. Prescott, who, while at Birkbeck College, made valuable translations of most of the foreign Papers referred to in the introduction.

ABSTRACT.

In this Paper an investigation is made of the distribution of temperature down a conductor conveying an electric current and at the same time moving uniformly through two fixed temperature sources. The effect of the Thomson heat on the distribution is seen to be exactly similar to the effect of a small impressed velocity. This result is applied to mercury to measure the Thomson effect by comparing the alteration of temperature $\Delta\theta_1$ at a point near the middle of the gradient caused by reversing a current of C amperes with the alteration of temperature $\Delta\theta_2$ at the same point due to a flow of mercury of m grammes per second. It is shown that, without any approximation as to emissivity loss or magnitude of Joulian heat, $2C\sigma/ms = \Delta\theta_1/\Delta\theta_2$, where s is the specific heat of mercury and σ the specific heat of electricity. Working with currents of from 4 to 9 amperes and with flows of different magnitudes—but never exceeding 1 cm. per hour—consistent values of σ are obtained, the value at 61°C. being $-1{\cdot}52\times10^{-6}$ calories per degree Centigrade per coulomb. The thermo-junctions, which were of iron and constantan, were fused through the glass tubes with inappreciable distortion.

DISCUSSION.

The PRESIDENT stated that the Paper dealt with a difficult problem and gave an adequate and promising method of measuring the Thomson effect, but he queried whether it was justified to assume the velocity of flow was constant over the cross-section of the tube.

Dr. A. GRIFFITHS stated that it was not assumed that the velocity over the cross-section was constant, but only that the temperature was constant, which on account of the extreme slowness of the flow would be justified. The author performed one experiment when the flow was stopped and obtained the same difference in temperature on reversing the current.

Dr. W. E. SUMPNER pointed out that it was not realised how extremely slow the flow was—something of the order of 1 cm. per hour.

Prof. C. H. LEES was struck with the ingenuity of the method. There were, however, a number of small corrections to be considered, such as the heat transmitted through the glass. More accurate knowledge of the thermoelectric phenomena in liquids was urgently needed.

Mr. R. S. WHIPPLE inquired how the iron and constantan wires were fused into the glass tube to which the Author replied.

V. *An Improved Joule Radiometer and its Applications.* By F. W. Jordan, *A.R.C.S.*, *B.Sc.*

Received October 21, 1912. Read November 8, 1912.

I have already described a simple Joule apparatus* adapted to show the existence of the Peltier and Thomson effects. The tube and partition in that apparatus possessed a considerable thermal capacity and were made of bad thermal conductors. The time of attaining temperature-equilibrium was consequently excessive, and its extreme sensibility to stray heat caused the zero to creep in such a way as to render the apparatus unsuitable for exact measurements. The following apparatus was designed to eliminate the necessity of elaborate thermal insulation and to give a steady zero and a fairly high sensibility.

A brass tube A (Fig. 1), 4 cm. long, 1·6 cm. diameter and 0·2 cm. thick, was divided longitudinally into two compartments by a copper plate B, 0·6 mm. thick. Two rectangular gaps were cut in diagonally opposite corners of the partition and two sector-shaped discs of copper, *cc*, were soldered to the edges of the gaps to form the horizontal sides of the channels for the current of air between the compartments. The partition B and the discs *cc* were soldered to the inner surface of the brass tube A. Two light mica vanes, *dd*, each 7 mm. by 6·5 mm. and total weight 1·4 mgm., were fixed with a little shellac to a fine glass stem *e*. The distance between the vanes and the other dimensions were arranged so that when suspended the clearance between the edges of the vanes and the sides of the channels was about 1 mm. A small silvered glass mirror *m*, 3·5 by 2·5 by 0·2 mm., was attached to the glass stem *e*, and the whole system was suspended by a quartz fibre 9 cm. long and 0·004 mm. diameter about. The flanged ends of the brass tube A and the brass cover plates DD were ground plane and screwed together tightly. The glass tube E carrying the torsion head was fitted over a short brass tube fixed centrally to the upper cover plate. The motion of the vanes was limited by stops to about 20 degrees. The air in the interior of the apparatus was thus practically sealed from communication with the atmosphere and shielded from stray heat by the brass

* "Nature," May 18, 1911, p. 380.

enclosure. The apparatus was fitted inside a concentric brass tube and the whole was mounted on a levelling stand. More elaborate thermal insulation would probably be required in the neighbourhood of intense sources of heat and also for a more sensitive instrument.

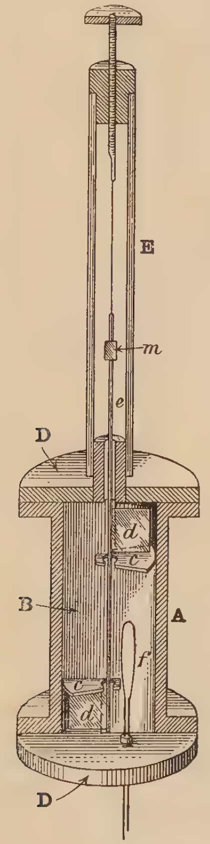


FIG. 1.

The sensibility of an instrument of this type may be expressed by the deflection in millimetres on a scale at a distance of a metre from the mirror produced by the absorption or evolution of heat at the rate of a microwatt in one of the compartments. This was determined by passing a measured current through a known resistance in one of the compartments.

The ends of a short length of No. 47 eureka wire f were soldered to two leads g of No. 36 copper wire. The insulated leads g were fastened together, and to the inside of a narrow glass tube with paraffin wax. The glass tube was passed through a circular aperture in the lower cover plate and sealed with chatterton compound ; a pair of dummy leads were similarly mounted in the other compartment to compensate any gain or loss of heat along the leads to the resistance loop.

The position of the spot of light on the scale was steady a few minutes after the apparatus had been levelled and adjusted. Bringing the hand quite near to the outer brass tube had no appreciable effect, and the zero remained steady to within a fraction of a millimetre. The motion of the vanes was excessively damped, and the period of swing was much greater than the undamped natural period.

The deflections produced by measured currents through the resistance loop were very nearly proportional to the squares of the currents. This agreed with the results I have obtained with other instruments of this type. In the case of the full scale deflection the spot of light became steady 40 seconds after the current was started through the resistance. On breaking the current the spot of light returned to the original zero in about the same interval of time. This interval of time could be reduced by diminishing the air resistance to the motion of the vane. The latter could be effected by increasing the clearance around the edges of the vanes. The following figures give some idea of the sensibility of the instrument.

A deflection of 122 mm. on a scale at a distance of a metre from the mirror was produced by a current of 4·75 milliamperes through the resistance of 10·5 ohms. This gives a sensibility of 0·52 mm. per microwatt. The small deflection of about 1 mm. produced by 0·5 milliampere could be repeatedly observed. The range of the instrument could be extended to measure feeble alternating currents of the order of 10^{-4} amperes by using a heater of 1,000 ohms, and the least detectable current in this case would be 5×10^{-5} amperes.

Alternating currents producing more than the full scale deflection could be conveniently and accurately measured by the compensation methods of H. L. Callendar* and K. Angström.† In Callendar's method the heat from the heater would be absorbed by the current through a thermo-junction in the

* Phys. Soc. "Proc." Vol. XXIII., March 17, 1911.

† Phys. "Zeit.," p. 685, 1905.

same compartment. In Ångström's method the heat would be balanced by the heat from a similar resistance in the other compartment. A calibration with direct currents would be necessary in every case to measure alternating currents.

Although I have not yet tried it, the instrument may be adapted to measure the heat given out by small quantities of radium. A calculation shows that 1 mgm. of radium should give a deflection of 50 mm. on the scale.

To measure radiant heat it would be sufficient for most purposes to make a small window of rock salt or fluorite of about 50 sq. mm. in the side of the brass tube A and direct the radiant heat on to a thin metal disc supported centrally by a fibre in one of the compartments. The rate of absorption of heat by the disc could be measured by Callendar's method. The indications of the instrument could thus be made independent of radiant heat from directions other than those limited by a tube directed towards the window and the receiving disc.

Previous to this I had constructed on similar lines an apparatus in which the tube and partition were made of thin mica and 12 cm. long. This larger instrument was eleven times as sensitive as the one described here, but owing to the difficulty of thermally insulating the apparatus it was abandoned. There seems to be no reason why that sensibility should not be attained again in an improved form of instrument.

In the original Joule apparatus the radiation from an external source was absorbed by the badly conducting glass walls and partition of cardboard. The convection current of air was thus formed near the inner surface of the compartment and attained a steady velocity when temperature equilibrium had been established. The latter state would only be complete after a considerable interval of time, owing to the large thermal capacity and small thermal conductivity of the enclosure. In the modified apparatus the heat radiated to the walls of a compartment is practically ineffective in producing a convection current, and thus one cause of its slowness of action and unsteadiness of zero has been removed.

Since the construction of this experimental form of radiometer a compensation method of measuring the Thomson effect has occurred to me. The present apparatus is on too small a scale and would have to be considerably modified in order to be suitable for this measurement. The following is an outline of the method.

A thin wire of the metal AB (Fig. 2) is attached with solder

or an electro-deposit of the same metal to the thick leads D and E, also of the same metal. These leads are insulated from, and led through, the lower cover plate of the radiometer. Two thin wires of different metals, OF and OG, are attached to wire AB to form a thermo-junction, O, for the measurement of the temperature at the centre of AB. One of these wires, OG, should be of the same metal as AB. Two small plates of mica, perforated to slip over AB, serve to separate the parts of the wire to the right and left of the thermo-junction and also to form a continuation of the partition of the radiometer.

Let an alternating current, C, or a direct current reversed at regular short intervals be passed through AB. The resistances of the two halves of the wire may be adjusted by electro-deposition, so that the convection currents on either side of the

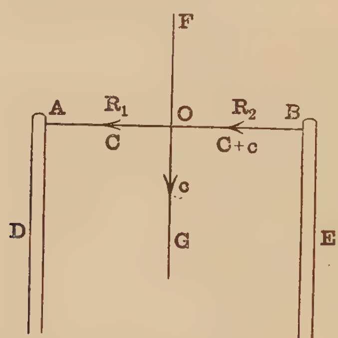


FIG. 2.

partition cause only a small deflection of the vane. This adjustment need not be made exactly, as balance can be effected by passing a current through an auxiliary heating coil. The fractions of the quantities of heat evolved by the halves of the wire, which are effective in producing the convection currents, will not in general be the same.

Let k_1 and k_2 be these fractions for the halves OA and OB respectively.

Let R_1 and R_2 be the resistances of the halves OA and OB respectively.

Let q be the rate of generation of heat by auxiliary coil, plus that which corresponds to small deflection of vane.

Then $$k_1C^2R_1 - k_2C^2R_2 = q.$$

A strong direct current, C, is then passed through the wire to

raise its temperature to about 100 deg. The cooling in one half of the wire due to the Thomson effect is compensated by passing a small current, c, in the opposite direction through the other half and the lead OG. The reverse holds for any heating due to Thomson effect. An approximate calculation shows that, by suitably dimensioning the wires, the current c can be made as small as 0·002C. Thus the Thomson and Joule effects in the lead OG can be neglected.

On changing from alternating to direct current the resistances will, owing to the Thomson effects, undergo small variations. In the following treatment, for the sake of clearness, it is assumed that the figure applies to the case where the Thomson effect is positive.

Let σ=Mean Thomson coefficient in volts per degree,
C=Current through BA,
c=Compensating current through OB,
t=Difference of temperature between the centre O and the leads D and E as determined with thermo-junction at O and others on leads D and E,
δR_1 and δR_2=Variations of the resistances R_1 and R_2 respectively.

Let the compensating current c be adjusted to give the same deflection as formerly. Then

$$k_1\{C^2(R_1+\delta R'_1)+\sigma Ct\}-k_2\{(C+c_1)^2(R_2+\delta R'_2) -\sigma(C+c_1)t\}=q. \quad (1)$$

The assumption that k_1 and k_2 for the Joulean heat are the same as for the Thomson heat is justifiable in the case where (1) there is a steep temperature gradient from the centre towards the end of wire AB, and (2) the greater part of the heat from wire AB is dissipated within the radiometer. Both these conditions can be nearly fulfilled.

Let C be reversed and compensation effected by adjusting c from B to O. Then

$$k_1\{C^2(R_1-\delta R''_1)-\sigma Ct\}-k_2\{(C-c_2)^2(R_2-\delta R''_2) +\sigma(C-c_2)t\}=q. \quad (2)$$

There remains on subtracting (2) from (1) and neglecting small terms

$$k_1C^2\delta R_1-k_2C^2\delta R_2+2k_1\sigma Ct+k_2\sigma t\{(C+c_1) +(C-c_2)\}-2k_2C(c_1+c_2)R_2=0, \quad (3)$$

where $\delta R_1=\delta R'_1+\delta R''_1$ and $\delta R_2=\delta R'_2+\delta R''_2$.

Let the compensating current c be now passed through the other half OA of the wire AB. Then similarly

$$k_2C^2\delta R'''_2-k_1C^2\delta R'''_1+2k_2\sigma Ct+k_1\sigma t\{(C+c_3)+(C-c_4)\}-2k_1C(c_3+c_4)R_1=0. \quad (4)$$

It may be assumed that $\delta R_1-\delta R'''_1=\delta R_2-\delta R'''_2$. Hence addition of (3) and (4) and division all through by C gives

$$4\sigma t(k_1+k_2)-\frac{(c_1+c_2+c_3+c_4)}{2}(R_1+R_2)(k_2+k_3)=0. \quad (5)$$

To obtain this equation it has been assumed that (1) (c_3-c_4) and (c_1-c_2) may be neglected in comparison with C, and (2) k_1 very nearly equals k_2, and (3) R_1 very nearly equals R_2.

Thus $$\sigma=\frac{(c_1+c_2+c_3+c_4)(R_2+R_1)}{8t} \quad . \quad . \quad . \quad . \quad (6)$$

To measure t two additional thermo-junctions would be attached to leads D and E, preferably near the cover plate of the radiometer. The resistance (R_2+R_1) would be the total resistance of wire AB, and the parts of the leads D and E between the extreme thermo-junctions. This resistance could be measured with sufficient accuracy by the aid of a potentiometer or a voltmeter.

All the quantities on the right of equation (6) are measurable to a sufficient degree of accuracy for the determination of the Thomson coefficient. It is very difficult to separate the Thomson effect in a metal from other parasitic E.M.F.s arising from possible inequalities in the composition and crystallographic structure of the metal. The latter is more especially true of those metals in which the Peltier and Thomson E.M.F.s vary with direction through the characteristic crystal of the metal. Hence in many cases too much reliance cannot be placed on the final result.

SOUTH-WESTERN POLYTECHNIC, CHELSEA, S.W.

ABSTRACT.

The first part of the Paper relates to improvements which have been made in order to convert the original Joule convection apparatus into an instrument for the exact measurement of small steady rates of evolution or absorption of heat. These improvements consisted in (1) replacing the badly conducting glass enclosure and cardboard partition by others made of brass and copper respectively; (2) replacing the uncertain and variable magnetic control of the movement of the vane in Joule's apparatus by the elastic control of a quartz fibre; (3) shaping the channels, in which the vanes moved, so that the angular deflection of the vanes was proportional to the rate of evolution of heat; (4) reducing the size, so that a more uniform

temperature of its various parts could be easily maintained by (5) placing the radiometer within a concentric brass tube to exclude all extraneous heat excepting that which might be directed through apertures in its side towards the radiometer.

The sensibility of the instrument was measured by passing a current through a resistance loop in one of the compartments of the partitioned tube, and found to be equal to 0·52 mm. per microwatt, as measured on a scale at a distance of 1 metre from the mirror. Thus the instrument may be used for the measurement of feeble oscillating currents, it being about as rapid as a Duddell milliammeter.

To convert the apparatus into an instrument for the measurement of radiant heat it is suggested that the radiant heat be directed through a small rock salt or fluorite window in the side of a compartment on to a thin blackened metal disc supported centrally by a badly conducting fibre within the compartment.

Its use for the quick measurement of the heat given out by radium is also suggested.

It is suggested that small steady rates of evolution or absorption of heat might be measured by the compensation methods of Callendar or Ångström.

The second part of the Paper relates to a suggested method of measuring the Thomson effect with this radiometer. The method hinges on an experiment described by the author in "Nature," May 18, 1911, p. 380. In that apparatus the halves of a thin wire on either side of the partition are heated by the passage of an alternating current through thicker leads of the same metal. The Joule effects are compensated very nearly by an electro-deposit of the same metal, by scraping the thin wire, or by an auxiliary heating coil. The substitution of a direct current for the alternating current causes a slight heating in one and a cooling in the other half of the wire. The heating or cooling due to the Thomson effect in one half of the wire is compensated by passing a small measured current in the proper direction through the other half of the wire. This small current is passed through a thin lead of the same metal attached to the centre of the thin wire, and may be adjusted in four different ways. The temperature difference between the centre of the wire and the thick leads is measured with suitably attached thermojunctions.

The Thomson coefficient is expressible in terms of measurable quantities, and is equal to the product of the mean compensating current and the mean resistance of the halves of the wire divided by the temperature difference between the centre of the thin wire and its thick leads.

DISCUSSION.

Dr. W. H. Eccles stated that he had worked a good deal with other forms of convection instruments. The better-known type consisted of a helix of wire which was caused to rotate by the draught up the tube. Forbes in 1890 patented a convection galvanometer with a screw propeller placed in the draught tube over the heater. He had developed this by using a fine paper screw propeller suspended by a quartz fibre, and used it for measuring small oscillatory currents, though his old instrument was 50 times less sensitive than Mr. Jordan's.

The Author stated that Crookes in 1887 had used vanes set at 45° in a tube to measure convection currents.

VI. *Note on the Attainment of a Steady State when Heat Diffuses along a Moving Cylinder.* By MISS A. SOMERS, *B.A.*

COMMUNICATED BY DR. A. GRIFFITHS, BIRKBECK COLLEGE.

RECEIVED OCTOBER 8, 1912. READ NOVEMBER 8, 1912.

IN experiments aiming at the determination of thermal conductivity, carried out by Mr. Nettleton, and described in the "Proceedings of the Physical Society of London," Vol. XXII., April, 1910, a column of mercury having its ends at fixed temperatures is kept in steady longitudinal motion, and it is necessary to ascertain the time of attainment of a steady flow of heat along the column. This has been done by experiment, but the following suggests a method of calculating the time from purely theoretical considerations :—

Let K=the thermal conductivity of the material of the column,

v=the velocity of the column when moving from the cooler to the hotter region,

ρ=the density of the material of the column,

s=the specific heat of the same,

θ=the temperature above the enclosure at any point within the column of distance x from the plane normal to the column through its cooler end, at a time t measured from the beginning of the experiment,

L=the length of the column between the points at fixed temperatures,

θ_L=the fixed temperature at the hotter end,

θ_0=the fixed temperature at the cooler end.

Let the initial temperature of the column be that of the enclosure.

Then, when the flow is from the cooler region, the equation which gives the temperature at any point within the column is

$$K\frac{d^2\theta}{dx^2}-v\rho s\frac{d\theta}{dx}=\rho s\frac{d\theta}{dt}.$$

The solution of this equation to satisfy the conditions of the experiment is

$$\theta-\theta_0=(\theta_L-\theta_0)\frac{1-e^{v\rho sx/K}}{1-e^{v\rho sL/K}}+\Sigma A_i e^{v\rho sx/2K}\sin\frac{i\pi x}{L}\cdot e^{-\left(\frac{v^2\rho s}{4K}+\frac{i^2\pi^2 K}{\rho s L^2}\right)t},$$

where i has every positive integral value in turn, and

$$Ai=\frac{2i\pi}{L^2\left(\frac{v^2\rho^2 s^2}{4K^2}+\frac{i^2\pi^2}{L^2}\right)}\left\{(-1)^i\theta_L e^{-\frac{v\rho s L}{2K}}-\theta_0\right\}\cdot$$

As the rate of flow of heat past any point in the cylinder is given by

$$K\frac{d\theta}{dx}-v\rho s(\theta-\theta_0),$$

it follows that the rate of flow across the hotter end of the cylinder is given by

$$(\theta_L-\theta_0)\cdot\frac{v\rho s}{e^{v\rho sL/K}-1}$$
$$+\frac{2K}{L}\Sigma\frac{i^2\pi^2K}{\frac{v^2\rho^2 s^2 L^2}{4K}+i^2\pi^2K}\left\{\theta_L-(-1)^i\theta_0 e^{v\rho sL/2K}\right\}e^{-\left(\frac{v^2\rho^2 s^2}{4K}+\frac{i^2\pi^2K}{L^2}\right)\frac{t}{\rho s}}$$

being $(\theta_L-\theta_0)\cdot\frac{v\rho s}{e^{v\rho sL/K}-1}$ when the steady state is attained.

Thus the formula gives the time of attainment of the steady state.*

It may be noted that a strictly analogous equation is applicable to the case of the diffusion of a salt in solution through a tube, both ends of which are kept at constant concentration, as described by Mr. Clack in the "Proceedings of the Physical Society of London," Vol. XXI., 1908, and Vol. XXIV., December, 1911. He has calculated the time required for the attainment of a steady rate of diffusion when the liquid is at rest, and concludes from experimental evidence that the actual small variable velocity of the liquid will not appreciably affect this time.

* Calculations based on these formulæ show that, with velocities of the magnitude suggested by Mr. Nettleton, an approximately steady state, involving less than 0·5° difference from the ultimate temperatures along the column, will be attained in just over two hours.

Assuming that the velocity is uniform, this conclusion is supported by calculation from a formula similar to the above, since the velocity in this case is of far lower order than the coefficient of diffusion.

ABSTRACT.

The Paper dealt with the case of a column of mercury moving with uniform speed between two fixed temperature sources. The differential equation for the temperature within the column was stated and its solution given, and it was shown how the time of attainment of a steady state could be obtained from the latter. The case of the diffusion of a salt in solution up a tube could be treated in the same manner.

DISCUSSION.

Dr. A. Griffiths asked if some Fellow would solve the problem when the velocity, instead of being constant, was a periodic function of the time.

Mr. B. W. Clack stated that Miss Somers had referred to his work on diffusion, and in such a slow phenomenon it was important to save as much time as possible. The velocity of the liquid down the diffusion tube referred to was in his experiments natural and not artificial, depending on the change in volume of the solution as it became less concentrated by the diffusion. This velocity was very slow. In his apparatus it was of the order 1 cm. in four months, and he felt justified in assuming that this would not materially alter the time required to attain the steady state. Experiments showed that this assumption was legitimate.

Mr. R. Appleyard drew some analogies between the differential equation used and that for the flow of electricity along conductors.

VII. *The Thermomagnetic Study of Steel.* By S. W. J. Smith, *M.A., D.Sc., Assistant Professor of Physics, Imperial College of Science.*

Received October 21, 1912. Read November 8, 1912.

Among the phenomena exhibited by magnetic materials, one recurs so frequently that it may reasonably be looked for in any substance whose magnetic properties are still incompletely known.

This phenomenon appears in curves plotted to show corresponding values of the permeability μ and the temperature θ for different values of H, the magnetising field. For each particular field there is in general a temperature at which the permeability is a maximum. When the field strength is comparatively large the maximum is not very pronounced and the $\mu\theta$ curve is concave to the axis of θ over a wide range. As the field strength is reduced, however, the maximum becomes (within certain limits) more and more clearly defined. The range over which the curve is concave to the axis of temperature becomes ultimately only a few degrees. Simultaneously the temperature of maximum permeability approaches the critical temperature of the substance.

These characteristic variations constitute the phenomenon referred to above. They suggest a method of exhibiting, with the maximum of clearness, the co-existence of different magnetic constituents in the same material. The object of this communication is to illustrate this method by showing how strikingly the presence of carbide of iron in steel lends itself to demonstration by its aid.

It has been shown in an earlier Paper* that the permeability of this carbide is relatively very small above 230°C. The measurements were therefore restricted to temperatures between that of the air and 250°C. They were made by the ballistic method upon a sample of the nearly pure carbon steel kindly given to me by Prof. Arnold, and stated to contain the following percentages of elements other than iron: C=0·85, Si=0·05, Mn=0·06, S=0·03 and P=0·02. The material was supplied in the form of carefully annealed

* "Proc." Phys. Soc., Vol. XXIV., 1911, p. 64.

rods half an inch in diameter. From one of these a tube 5 cm. long was constructed, having an inductive cross-sectional area of nearly 1 sq. cm.

It was found by trial that cotton immersed in melted paraffin wax retains its insulating power sufficiently well, for the purposes of magnetic measurements, up to 250°C., although it chars and becomes defective at much lower temperatures when heated in air. The primary and secondary coils were therefore constructed of cotton-covered wire and the "ring" was suspended, by the leads from the coils, within a test tube containing more than enough wax to cover it completely. Heat was conveyed to the test tube through a thick containing tube made of copper. The temperature of the ring was measured by means of a calibrated copper-constantan couple placed within it and also by means of a mercury thermometer in contact with its outer surface.

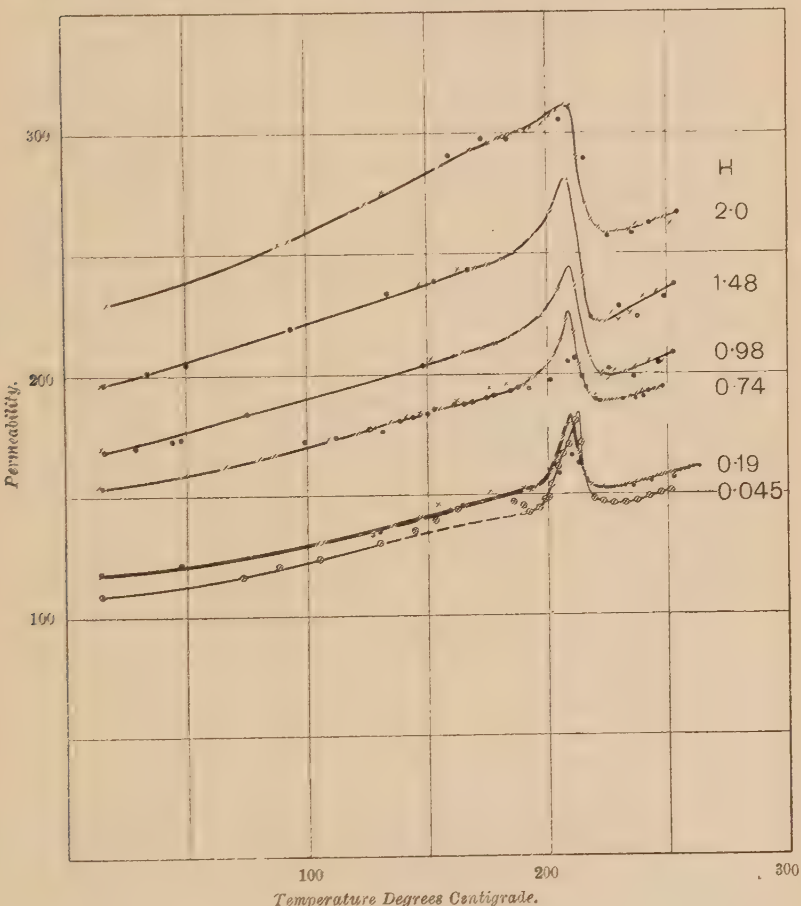
Some of the results of the experiments are shown in the figure on page 79 in which the abscissæ are temperatures and the ordinates are the approximate permeabilities, in absolute units, corresponding with the values of H marked opposite the curves. The points denoted by crosses represent observations taken during cooling. Only a few observations (represented by dots) were taking during heating, owing to the relatively greater difficulty of temperature regulation in that case.

The curves present various features of interest which it seems inadvisable to discuss in detail until further experiments upon this and upon other steels containing different percentages of carbon have been completed.

It seems obvious, however, that the contribution of the carbide towards the apparent permeability of the material as a whole becomes more and more restricted in its range as the strength of the magnetising field is lowered. Until, when the field is below 0·2 gauss, the even sweep of the curve between 20°C. and 250°C. suffers no considerable disturbance except over some tens of degrees in the neighbourhood of the critical temperature of the carbide. To show this the more clearly in the figure the curve for $H = 0{\cdot}19$ C.G.S. is drawn more boldly than the rest. Attention may also be drawn to the curve for $H = 0{\cdot}045$, the weakest field in which measurements could conveniently be taken with the apparatus used. In this case the maximum permeability occurs in the neighbourhood of 213°C. and is nearer the critical temperature than any of the other maxima. The $\mu\theta$ curve also descends more

steeply beyond the maximum than in any of the other cases. Several of the curves—this one in particular—show evidence of a secondary maximum which may possibly be due to some unsuspected constituent.

It is not, of course, to be supposed that the fields given in the figure are the actual fields in which, for example, a ring of the



pure carbide would exhibit variations of permeability precisely similar to those just described. The carbide, as we have examined it, is embedded in a material of which the permeability is in general very different from its own. The effective

field within it will therefore be different in general from the average field within the steel as a whole and will, moreover, be different in different parts of the ring—unless the distribution of the carbide is exceptionally regular.

It is, however, unnecessary for the present purpose to lay stress upon this point. The immediate object is only to show that, by experiment, fields can be found which make the existence of a particular constituent very conspicuous.

When it is remembered that the existence of any definite carbide of iron has often been a subject of dispute amongst metallurgists and others, and even now rests mainly upon chemical analyses somewhat difficult to perform, it is surprising that purely physical methods of indicating its existence were not multiplied long ago.

It will be seen that the results given in this Paper explain the rounded outlines of the permeability-temperature curves given in Fig. 3 of the earlier Paper* in which fields stronger than **H**=2 were applied. They explain also why the temperature of the maximum permeability below 250°C. becomes lower as the field strength is raised. Similarly, they account for the shapes of the curves given in recent Papers by Maurain† and by Moir.‡ All of these data, to which may be added the results given in a still more recent Paper by Mr. Guild and the writer,§ can therefore be cited in proof of the existence of the same constituent in steels containing the most widely differing percentages of carbon. They show also that the relative amount of this constituent increases with the percentage of carbon, and it is quite possible that a method could be devised by which, from the thermo-magnetic properties alone, the percentage of carbon in any particular steel could be easily determined.

ABSTRACT.

Thermomagnetic measurements make it increasingly evident that the magnetic properties of steels are frequently those of mixtures of magnetic substances, each possessing characteristic properties, which contribute in a comparatively definite way to the properties of the material as a whole.

In the case of a simple ferromagnetic substance, magnetising fields can generally be found in which the permeability variation with tem-

* *Loc. cit.*, p. 66.
† "Ann. de Chim. et de Phys.," Vol. XX., 1910, pp. 353–389.
‡ "Proc." R.S.E., Vol. XXXI., 1911, pp. 505–516.
§ "Proc." Phys. Soc., Vol. XXIV., 1912, pp. 342–348.

perature is comparatively small except in the neighbourhood of the critical temperature. In such fields there is a very clearly marked peak in the permeability temperature curve for the substance. The explanation of this peak which the molecular theory affords is well known, and suggests that the phenomenon should be found common to all ferromagnetic substances. The immediate object of the present Paper is to show that it is exhibited by the carbide of iron (cementite) which exists in annealed carbon steels. For this purpose it is not necessary to isolate the carbide because, as shown in the Paper, the phenomenon is quite clearly discernible in the permeability temperature curves for the steel. The particular steel examined contained 0·85 per cent. of carbon. It was found that the fields necessary to evoke the comparatively sudden variations in the permeability of the carbide above described are small and such that the permeability variation of the iron present along with the carbide is slight in the neighbourhood of the critical temperature of the latter. The sudden gain and loss of permeability by the carbide as the temperature alters will be roughly equivalent to sudden removal and replacement of gaps in the magnetic circuit through the steel. They should therefore be attended by correspondingly sudden rise and fall of the apparent permeability of the material as a whole. This is found to be the case. There is a sharply marked peak near 210°C. upon the permeability temperature curve for the steel.

In the absence of measurements between 200°C. and 220°C. the peak would escape notice and it is for this reason, probably, that it has not been recorded before. It could scarcely be found by accident. The search for it was prompted by the considerations outlined above in conjunction with results obtained in earlier work with Messrs. White and Barker.

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